Year 13 Science: Simple harmonic motion

Name: _____ Date: _____

Try these without your notes first, then check the answer key.

Practice

1. Find the period of a simple pendulum 0.800 m long.

2. A 0.500 kg mass hangs on a spring of stiffness 20.0 N m^{-1}. Find the period.

3. An object oscillates in SHM with amplitude 5.00 cm and period 0.400 s. Find its maximum speed and maximum acceleration.

4. A 0.300 kg mass on a spring of stiffness 120 N m^{-1} oscillates with amplitude 0.0500 m. Find the total energy and the speed when y = 0.0300 m.

Exam-style question: a mass on a spring

A 0.250 kg mass hangs at rest on a spring of stiffness 40.0 N m^{-1}. It is pulled 6.00 cm below its equilibrium position and released.

5. Show that the angular frequency of the motion is 12.6 rad s^{-1}.

6. Calculate the maximum speed and the maximum acceleration of the mass.

7. Calculate the time after release when the mass first reaches 2.00 cm above equilibrium. A reference circle may help.

8. The top of the spring is then moved up and down by a hand at a frequency of 2.0 Hz. Explain, using forces and energy, why the mass oscillates with a very large amplitude.

Fix the mistake

9. Correct this statement: "$\cos^{-1}(-0.500)$ in degree mode gives 120, so t = 120 ÷ 3.927 = 30.6 s."

10. Correct this statement: "The acceleration is zero at the extremes because the object stops there."

11. Correct this statement: "Using $y = A \sin \omega t$ for a mass released from rest at the top."

12. Correct this statement: "A pendulum with a heavier bob or a bigger swing has a longer period."

Answer key (for parents and tutors)

1. Find the period of a simple pendulum 0.800 m long.
   $T = 2\pi\sqrt{(0.800 \div 9.81)} = 1.79$ s.

2. A 0.500 kg mass hangs on a spring of stiffness 20.0 N m^{-1}. Find the period.
   $T = 2\pi\sqrt{(0.500 \div 20.0)} = 0.993$ s.

3. An object oscillates in SHM with amplitude 5.00 cm and period 0.400 s. Find its maximum speed and maximum acceleration.
   $\omega = 2\pi \div 0.400 = 15.71$ rad s^{-1}; $v_{max} = A\omega = 0.0500 \times 15.71 = 0.785$ m s^{-1}; $a_{max} = A\omega^2 = 0.0500 \times 15.71^2 = 12.3$ m s^{-2}.

4. A 0.300 kg mass on a spring of stiffness 120 N m^{-1} oscillates with amplitude 0.0500 m. Find the total energy and the speed when y = 0.0300 m.
   $E = \frac{1}{2}kA^2 = \frac{1}{2} \times 120 \times 0.0500^2 = 0.150$ J; $\omega = \sqrt{(120 \div 0.300)} = 20.0$ rad s^{-1}, so $v = \omega\sqrt{(A^2 - y^2)} = 20.0 \times \sqrt{(0.0500^2 - 0.0300^2)} = 20.0 \times 0.0400 = 0.800$ m s^{-1}.

5. Show that the angular frequency of the motion is 12.6 rad s^{-1}.
   $\omega = \sqrt{(k \div m)} = \sqrt{(40.0 \div 0.250)} = \sqrt{160} = 12.65$ rad s^{-1}, which is 12.6 rad s^{-1} (Achieved).

6. Calculate the maximum speed and the maximum acceleration of the mass.
   $v_{max} = A\omega = 0.0600 \times 12.65 = 0.759$ m s^{-1}; $a_{max} = A\omega^2 = 0.0600 \times 160 = 9.60$ m s^{-2} (Achieved for one, Merit for both).

7. Calculate the time after release when the mass first reaches 2.00 cm above equilibrium. A reference circle may help.
   Taking up as positive, $y = -0.0600 \cos \omega t$; at $y = +0.0200$, $\cos \omega t = -0.333$, so $\omega t = 1.911$ rad and $t = 1.911 \div 12.65 = 0.151$ s. On the reference circle the phasor turns 90° to equilibrium and then a further $\sin^{-1}(0.0200 \div 0.0600) = 19.5°$, 109.5° in all (Merit for the method with one error, Excellence for 0.151 s).

8. The top of the spring is then moved up and down by a hand at a frequency of 2.0 Hz. Explain, using forces and energy, why the mass oscillates with a very large amplitude.
   The natural frequency is $f = 12.65 \div 2\pi = 2.01$ Hz, so the hand drives the system at its natural frequency. Each push from the hand is then in step with the motion, in the same direction as the velocity, so the driving force does positive work on every cycle. Energy is added faster than damping removes it, so the energy stored, $\frac{1}{2}kA^2$, builds up cycle after cycle and the amplitude grows very large until the energy lost per cycle equals the energy supplied: this is resonance (Excellence links the matched frequency, the in-step force and the energy balance).

9. Correct this statement: "$\cos^{-1}(-0.500)$ in degree mode gives 120, so $t = 120 \div 3.927 = 30.6$ s."
   ωt is in radians, so set the calculator to radians before any sin ωt or cos ωt.

10. Correct this statement: "The acceleration is zero at the extremes because the object stops there."
    The velocity is zero at the extremes, but the restoring force and acceleration are greatest there, $A\omega^2$.

11. Correct this statement: "Using $y = A \sin \omega t$ for a mass released from rest at the top."
    Choose the function from the start: from rest at +A use $y = A \cos \omega t$, from equilibrium use $y = A \sin \omega t$.

12. Correct this statement: "A pendulum with a heavier bob or a bigger swing has a longer period."
    $T = 2\pi\sqrt{(l \div g)}$ depends only on length and g, for small swings; mass and amplitude cancel.