Year 13 Science: Rotational motion

Name: _____ Date: _____

Try these without your notes first, then check the answer key.

Practice

1. A wind turbine turns at 15.0 revolutions per minute. Find its angular velocity and the speed of a blade tip 40.0 m from the hub.

2. A fan spinning at 25.0 rad s^{-1} slows uniformly to rest in 10.0 s. Find its angular acceleration and how many turns it makes.

3. A net torque of 0.900 N m acts on a wheel with rotational inertia 0.0450 kg m^2. Find its angular acceleration.

4. A skater spinning at 3.00 rad s^{-1} changes her rotational inertia from 4.00 kg m^2 to 1.60 kg m^2. Find her new angular velocity and her rotational kinetic energy before and after.

Exam-style question: the potter's wheel

A potter's wheel has a rotational inertia of 0.850 kg m^2. The motor brings it from rest to 12.0 rad s^{-1} in 3.00 s with constant angular acceleration.

5. Calculate the net torque on the wheel while it speeds up.

6. Calculate how many turns the wheel makes in those 3.00 s.

7. With the motor off and the wheel spinning freely at 12.0 rad s^{-1}, the potter drops a 4.00 kg disc of clay, radius 0.120 m, centrally onto it. The clay's rotational inertia is $\frac{1}{2}mr^2$. Calculate the new angular velocity.

8. Explain why the rotational kinetic energy of the system decreases when the clay lands, even though angular momentum is conserved.

Fix the mistake

9. Correct this statement: "1200 rpm is put straight into $\omega f = \omega i + \alpha t$ as $\omega = 1200$."

10. Correct this statement: "Angular momentum is conserved, so rotational kinetic energy is conserved too."

11. Correct this statement: "The torque is F × r, using the distance to where the force acts, even when the force is at an angle."

12. Correct this statement: "The calculator is in degree mode for the trig in a rotation problem."

Answer key (for parents and tutors)

1. A wind turbine turns at 15.0 revolutions per minute. Find its angular velocity and the speed of a blade tip 40.0 m from the hub.
   $\omega = 15.0 \times 2\pi \div 60 = 1.571$ rad s^{-1}, about 1.57 rad s^{-1}; $v = r\omega = 40.0 \times 1.571 = 62.8$ m s^{-1}.

2. A fan spinning at 25.0 rad s^{-1} slows uniformly to rest in 10.0 s. Find its angular acceleration and how many turns it makes.
   $\alpha = (0 - 25.0) \div 10.0 = -2.50$ rad s^{-2}; $\theta = \frac{1}{2}(25.0 + 0) \times 10.0 = 125$ rad, and $125 \div 2\pi = 19.9$ turns.

3. A net torque of 0.900 N m acts on a wheel with rotational inertia 0.0450 kg m^2. Find its angular acceleration.
   $\alpha = \tau \div I = 0.900 \div 0.0450 = 20.0$ rad s^{-2}.

4. A skater spinning at 3.00 rad s^{-1} changes her rotational inertia from 4.00 kg m^2 to 1.60 kg m^2. Find her new angular velocity and her rotational kinetic energy before and after.
   $\omega f = 4.00 \times 3.00 \div 1.60 = 7.50$ rad s^{-1}; Ek before $= \frac{1}{2} \times 4.00 \times 3.00^2 = 18.0$ J, after $= \frac{1}{2} \times 1.60 \times 7.50^2 = 45.0$ J.

5. Calculate the net torque on the wheel while it speeds up.
   $\alpha = 12.0 \div 3.00 = 4.00$ rad s^{-2}; $\tau = I\alpha = 0.850 \times 4.00 = 3.40$ N m (Achieved).

6. Calculate how many turns the wheel makes in those 3.00 s.
   $\theta = \frac{1}{2}(0 + 12.0) \times 3.00 = 18.0$ rad, and $18.0 \div 2\pi = 2.86$ turns (Achieved).

7. With the motor off and the wheel spinning freely at 12.0 rad s^{-1}, the potter drops a 4.00 kg disc of clay, radius 0.120 m, centrally onto it. The clay's rotational inertia is $\frac{1}{2}mr^2$. Calculate the new angular velocity.
   $I_{clay} = \frac{1}{2} \times 4.00 \times 0.120^2 = 0.0288$ kg m^2; with no external torque, $0.850 \times 12.0 = (0.850 + 0.0288) \times \omega f$, so $\omega f = 10.2 \div 0.8788 = 11.6$ rad s^{-1} (Merit for the method, Excellence with the clay's I included).

8. Explain why the rotational kinetic energy of the system decreases when the clay lands, even though angular momentum is conserved.
   Ek(rot) before $= \frac{1}{2} \times 0.850 \times 12.0^2 = 61.2$ J; after $= \frac{1}{2} \times 0.8788 \times 11.6^2 = 59.2$ J. When the clay lands it is not spinning, so the wheel's surface slides under it at first; friction between them gives the clay a torque that speeds it up and an equal and opposite torque that slows the wheel. These internal torques cancel, so angular momentum is conserved, but the sliding friction turns about 2.0 J of kinetic energy into heat, like an inelastic collision (Excellence links the internal torques to conservation of L and the friction to the energy loss).

9. Correct this statement: "1200 rpm is put straight into $\omega f = \omega i + \alpha t$ as $\omega = 1200$."
   Convert to rad s^{-1} first: $1200 \times 2\pi \div 60 = 125.7$ rad s^{-1}.

10. Correct this statement: "Angular momentum is conserved, so rotational kinetic energy is conserved too."
    L is conserved with no external torque, but Ek(rot) changes when I changes; pulling in raises it, a lump landing on a turntable lowers it.

11. Correct this statement: "The torque is F × r, using the distance to where the force acts, even when the force is at an angle."
    Use the perpendicular distance from the axis to the line of the force, or the force component at right angles to r.

12. Correct this statement: "The calculator is in degree mode for the trig in a rotation problem."
    Angular quantities are in radians, so set the calculator to radians for sin ωt and similar.