Vertex Academy

NCEA Level 2 Maths workbook

9 worksheets · answers at the back

Contents

1. Index laws, fractional indices and surds
2. The derivative: what the gradient function means
3. Probability: when to add and when to multiply
4. Distance, midpoint and gradient
5. Families of graphs
6. Arithmetic sequences and series
7. The sine and cosine rules
8. Sampling and the enquiry cycle
9. Simultaneous linear equations

NCEA Level 2 · AS 91261: Algebra Toolkit

Name: _____ Date: _____

Kia ora! This standard is all about the algebra machinery: simplifying with indices, factorising, and solving quadratic, exponential and log equations. Work through each section in order, check every answer against the worked solution and note the grade level you're aiming for.

Revision summary

- Index laws: $a^m \times a^n = a^{m+n}$, $a^m \div a^n = a^{m-n}$, $(a^m)^n = a^{mn}$, $a^0 = 1$, $a^{-n} = 1/a^n$, $a^{(1/n)} = \sqrt[n]{a}$.
- Logarithm definition: if $a^x = b$ then $\log_a b = x$. Log laws: $\log(xy) = \log x + \log y$, $\log(x/y) = \log x - \log y$, $\log(x^n) = n \log x$.
- Quadratic formula: for $ax^2 + bx + c = 0$, $x = (-b \pm \sqrt{(b^2 - 4ac)}) \div 2a$.
- Discriminant $\Delta = b^2 - 4ac$: $\Delta > 0$ two real roots, $\Delta = 0$ one repeated root, $\Delta < 0$ no real roots.
- Factorising: common factor first, then quadratics $x^2 + bx + c = (x + p)(x + q)$ where $p + q = b$ and $pq = c$; difference of two squares $a^2 - b^2 = (a - b)(a + b)$.
- Exponential equations: write both sides with the same base, then equate the powers.
- Log equations: combine to a single log, rewrite in index form, then solve, and always check answers against the domain (you can only take logs of positive numbers).

Key skills

1. Simplify $(3a^2b)^3 \times 2ab^2$.

2. Factorise $x^2 + 3x - 10$.

3. Solve $2^x = 32$.

4. Evaluate $\log_3 81$.

Achieved

5. Solve $x^2 - 5x + 6 = 0$ by factorising.

6. Simplify $(12x^5y^3) \div (4x^2y^5)$.

7. Use the quadratic formula to solve $2x^2 + 3x - 4 = 0$. Give exact answers.

8. Solve 3^(x+1) = 81.

Merit

9. A rectangular deck at a bach in Raglan has a length 3 m longer than its width. The area of the deck is 40 m². Form an equation and find the dimensions of the deck.

10. Write $3/(x-2) + 2/(x+1)$ as a single simplified fraction.

11. Solve $9^x = 27^(x-2)$.

Excellence

12. For what values of k does the equation $x^2 + kx + 4 = 0$ have two distinct real roots? Justify your answer.

13. Solve $\log_2 x + \log_2(x-2) = 3$, showing why any invalid solution must be rejected.

14. Prove that the difference between the squares of any two consecutive odd numbers is always a multiple of 8.

NCEA Level 2 · AS 91262: Calculus Methods

Name: _____ Date: _____

This standard is all about differentiation, integration, and using them to solve gradient, tangent, optimisation and motion problems. Work through the sections in order, build fluency first, then push into the Merit and Excellence reasoning.

Revision summary

- Differentiation: if $y = ax^n$ then $dy/dx = anx^{n-1}$. The derivative gives the gradient function.
- Gradient of a curve at $x = a$ is found by substituting a into dy/dx. Tangent line: $y - y_1 = m(x - x_1)$.
- Anti-differentiation (integration): $\int ax^n\,dx = ax^{n+1}/(n+1) + c$ $(n \neq -1)$. Always add $+ c$ for an indefinite integral; find c using a known point.
- Increasing where $dy/dx > 0$, decreasing where $dy/dx < 0$. Turning points where $dy/dx = 0$.
- Second derivative test: $d^2y/dx^2 > 0 \rightarrow$ minimum, $d^2y/dx^2 < 0 \rightarrow$ maximum.
- Kinematics: differentiate displacement $s \rightarrow$ velocity $v \rightarrow$ acceleration a. Integrate $a \rightarrow v \rightarrow s$. At rest means $v = 0$.
- Optimisation: write the quantity as a function of one variable, differentiate, set derivative $= 0$, solve, and check it is a max or min.
- Units: gradient/rate = change in y per unit x; velocity in m s^{-1}, acceleration in m s^{-2}.

Key skills

1. Differentiate $y = 4x^3 - 5x^2 + 7x - 2$.

2. Find $\int (6x^2 + 4x - 3)\,dx$.

3. Find the gradient of the curve $y = x^2 - 6x + 5$ at the point where $x = 4$.

4. A ball's height is $h = 20t - 5t^2$ metres. Find its velocity function.

Achieved

5. Find the equation of the tangent to $y = x^2 - 3x + 1$ at the point where $x = 2$.

6. The gradient function of a curve is $dy/dx = 3x^2 - 4x$. The curve passes through $(1, 2)$. Find y as a function of x.

7. Find the coordinates of the turning point of $y = x^2 - 8x + 10$.

8. A drone rises so its height is $h = t^3 - 6t^2 + 9t$ metres, where t is in seconds. Find its velocity when $t = 1$ s.

Merit

9. A rectangular vegetable garden in Avondale is to be fenced against a straight wall, so only three sides need fencing. There are 60 m of fencing available. Find the dimensions that give the maximum area.

10. A curve has gradient function $dy/dx = 6x - 12$. Find the x-values where the curve is decreasing, and find where the curve has a turning point.

11. A particle moves so its velocity is $v = 3t^2 - 12t + 9$ m s^{-1}. It starts at position $s = 5$ m when $t = 0$. Find its displacement function and the times when it is at rest.

12. The tangent to the curve $y = x^2 + kx - 4$ at the point where $x = 1$ has gradient 5. Find k, then find the equation of that tangent.

Excellence

13. A cylindrical rainwater tank (no lid) is to hold a fixed volume V. Show that the surface area of metal used is least when the height equals the radius.

14. A curve $y = ax^3 + bx^2$ has a turning point at $(2, -4)$. Find a and b, justifying your reasoning.

15. A ball is thrown upward with height $h = ut - 5t^2$ metres (u is the initial speed in m s^{-1}). Prove that the maximum height reached is $u^2/20$ metres, regardless of u.

NCEA Level 2 · AS 91267: Probability Methods

Name: _____ Date: _____

Hi! This standard is all about working out chances using tables, trees, risk and the normal distribution. Work through each section in order, check your answers against the workings, and don't skip the Excellence justifications, that's where the top marks live.

Revision summary

- Probability of an event: P(A) = (favourable outcomes) ÷ (total outcomes), always between 0 and 1.
- Two-way tables: read off joint totals; row/column totals give marginal probabilities.
- Conditional probability: P(A given B) = P(A and B) ÷ P(B). On a tree, follow the relevant branch.
- Tree diagrams: multiply along branches, add across branches. Branch probabilities from each node sum to 1.
- Risk (relative frequency) = (number with outcome) ÷ (total in group). Relative risk = risk of group A ÷ risk of group B.
- Relative risk > 1 means higher risk than the baseline group; = 1 means equal risk; < 1 means lower risk.
- Normal distribution: symmetric, bell-shaped about the mean μ. Standardise with $z = (x - \mu) \div \sigma$, then use tables/calculator for the area (probability).
- Roughly 68% of data lies within 1 σ of the mean, 95% within 2 σ, 99.7% within 3 σ.

Key skills

1. A bag holds 5 red, 3 blue and 2 green marbles. One marble is drawn at random. Find P(blue).

2. In a survey of 200 people, 120 own a bike and of those 45 also own a scooter. Find P(owns a scooter given owns a bike).

3. A spinner lands on 'win' with probability 0.4. It is spun twice. Draw/consider a tree and find P(win then win).

4. Heights are normally distributed with mean $\mu = 170$ cm and standard deviation $\sigma = 8$ cm. Find the z-score for a height of 182 cm.

Achieved

5. At Avondale College, 300 students were asked if they walk to school. The two-way table shows: Junior students – 90 walk, 60 do not; Senior students – 50 walk, 100 do not. Find the probability a randomly chosen student walks to school.

6. A café finds 70% of customers order coffee. Of those who order coffee, 30% also order a muffin. Draw a tree for the first branch and find P(orders coffee and a muffin).

7. The time to fill a petrol tank is normally distributed with mean 4.5 minutes and standard deviation 0.8 minutes. Find the probability a fill takes less than 5 minutes.

8. In a town, 40 of 500 dog owners reported their dog had fleas, while 12 of 300 cat owners reported fleas. Calculate the risk of fleas for dog owners and for cat owners.

Merit

9. A factory has two machines. Machine A makes 60% of items with a 3% defect rate; Machine B makes 40% with a 5% defect rate. An item is chosen at random and found to be defective. Find the probability it came from Machine A.

10. Using the flea data (dogs: risk 0.08, cats: risk 0.04), calculate the relative risk of fleas for dogs compared with cats and interpret it in context.

11. Bags of flour are filled to a mean of 1500 g with standard deviation 12 g, normally distributed. Bags under 1485 g are rejected. In a batch of 800 bags, estimate how many are rejected.

Excellence

12. A medical test for a condition is 96% accurate for people who have it (true positive) and gives a false positive 5% of the time. In the population, 2% of people have the condition. A person tests positive. Find the probability they actually have the condition, and comment on why the result may seem surprising.

13. Two events A and B satisfy P(A) = 0.5, P(B) = 0.4 and P(A and B) = 0.2. Determine whether A and B are independent, and prove your conclusion using probability rules.

14. The lifetimes of two brands of LED bulb are normally distributed. Brand X: mean 20000 h, σ = 2000 h. Brand Y: mean 22000 h, σ = 3500 h. A retailer wants bulbs lasting at least 18000 h. Which brand gives a higher chance of meeting this, and by how much? Justify whether a larger mean always guarantees a better result here.

NCEA Level 2 · AS 91256: Lines, Points & the Coordinate Plane

Name: _____ Date: _____

This standard is all about working with points and straight lines on the number plane, distances, midpoints, gradients, equations of lines, and parallel/perpendicular relationships. Work through each section in order; check your working against the answers, and note the 'evidence' hints if you're aiming for Merit or Excellence.

Revision summary

- Distance between $A(x_1,y_1)$ and $B(x_2,y_2)$: $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$.
- Midpoint of AB: $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.
- Gradient: $m = \frac{y_2-y_1}{x_2-x_1}$ = rise ÷ run.
- Equation of a line: $y = mx + c$, or $y - y_1 = m(x - x_1)$ given a point and gradient.
- Parallel lines have equal gradients ($m_1 = m_2$).
- Perpendicular lines have gradients that multiply to −1 ($m_1 \times m_2 = -1$), so $m_2 = -1/m_1$.
- To find where two lines meet, solve their equations simultaneously.
- Perpendicular bisector of AB: passes through the midpoint of AB with gradient −1/(gradient of AB).

Key skills

1. Find the distance between A(1, 2) and B(7, 10).

2. Find the midpoint of P(−3, 5) and Q(7, −1).

3. Find the gradient of the line through C(2, 3) and D(6, 11).

4. Write the equation of the line with gradient 3 that passes through (2, −1).

Achieved

5. Find the equation of the straight line passing through the points (1, 2) and (4, 11).

6. A line has equation $y = 2x + 5$. Find the equation of the line perpendicular to it that passes through (4, 1).

7. Find the equation of the line parallel to $y = -3x + 2$ that passes through (2, 5).

8. On a walking-track map (units in km), a café is at (2, 3) and a lookout is at (8, 11). How far apart are they in a straight line?

Merit

9. A triangle has vertices A(−2, 1), B(4, 3) and C(2, 9). Show that the triangle has a right angle, and state where it is.

10. Two straight roads are modelled by $y = 2x - 3$ and $x + y = 9$. Find the coordinates of the point where the roads cross.

11. Find the equation of the perpendicular bisector of the line segment joining A(1, 2) and B(5, 8).

Excellence

12. A quadrilateral has vertices A(1, 1), B(5, 3), C(7, 7) and D(3, 5). Prove that ABCD is a rhombus.

13. A stream through a park follows the line $y = 2x$ (the park entrance is the origin). A pōhutukawa tree stands at T(5, 0). Find the shortest distance from the tree to the stream, and the coordinates of the nearest point on the stream. Justify why this is the shortest distance.

14. Two lines are given by $y = (k - 1)x + 3$ and $2x + 4y = 7$, where k is a constant. Find the value of k that makes the lines perpendicular, and explain your reasoning.

NCEA Level 2 · AS 91257: Graphs & Graphical Methods

Name: _____ Date: _____

This standard is all about drawing, reading and transforming graphs, lines, parabolas, exponentials and more, and using them to model real situations. Work through the sections in order and check every step against the worked answers.

Revision summary

- Line: $y = mx + c$, where m = gradient = rise/run and c = y-intercept. Parallel lines share m; perpendicular lines have gradients that multiply to −1.
- Parabola: $y = a(x - h)^2 + k$ has vertex (h, k). $y = a(x - p)(x - q)$ has x-intercepts at p and q; axis of symmetry midway between them.
- Transformations of $y = f(x)$: $y = f(x) + k$ shifts up k; $y = f(x - h)$ shifts right h; $y = af(x)$ stretches vertically by factor a; $y = -f(x)$ reflects in the x-axis.
- Exponential: $y = a \cdot b^x$ has a horizontal asymptote ($y = 0$ unless shifted); $b > 1$ grows, $0 < b < 1$ decays; y-intercept is a.
- Intercepts: set $y = 0$ for x-intercepts (roots), set $x = 0$ for the y-intercept.
- Asymptotes: lines a curve approaches but never reaches, key for exponential and reciprocal ($y = 1/x$) graphs.
- To model: pick the graph type that matches the pattern (constant rate → linear, constant multiplier → exponential, single turning point → quadratic), then fit features to the data.
- Solve simultaneously by finding where two graphs intersect, equal y-values.

Key skills

1. State the gradient and y-intercept of the line $y = 3x - 5$, then find the x-intercept.

2. Write the coordinates of the vertex of $y = (x - 4)^2 + 2$ and state whether it is a maximum or minimum.

3. Describe the transformation that takes $y = x^2$ to $y = x^2 - 6$.

4. For the exponential $y = 2^x$, state the y-intercept and the equation of the horizontal asymptote.

Achieved

5. A water tank at a bach near Muriwai starts with 500 L and is filled at a constant 40 L per minute. Write an equation for the volume V after t minutes and find the volume after 12 minutes.

6. Sketch $y = (x + 1)(x - 5)$ by finding its x-intercepts, axis of symmetry and vertex.

7. The population of a rabbit colony is modelled by $P = 30 \times 1.5^t$, where t is in years. Find the starting population and the population after 3 years.

8. A line passes through (0, 4) and (2, 10). Find its equation in the form $y = mx + c$.

Merit

9. A drone is launched from a 5 m platform. Its height (m) after t seconds is $h = -5t^2 + 20t + 5$. Find the maximum height reached and when it hits the ground.

10. The graph of $y = x^2$ is transformed to $y = 2(x - 3)^2 - 4$. Describe the sequence of transformations, then state the coordinates of the new vertex and the y-intercept.

11. Two courier companies charge for a delivery. FastVan charges $8 plus $2 per km; QuickGo charges a flat $20. Write an equation for each, find the distance where the costs are equal, and state which is cheaper for a 10 km trip.

Excellence

12. A parabola has x-intercepts at $x = -2$ and $x = 6$ and passes through $(2, -32)$. Find its equation in the form $y = ax^2 + bx + c$, and justify why the vertex must lie on $x = 2$.

13. An exponential model $y = a \cdot b^x$ passes through $(0, 12)$ and $(2, 3)$. Find a and b, and explain what value of b tells you about the situation and the behaviour of the graph.

14. Prove that the line $y = x + k$ is tangent to the parabola $y = x^2$ only when $k = -1/4$, and state the point of tangency.

NCEA Level 2 · AS 91258: Sequences & Series in Action

Name: _____ Date: _____

This standard is all about spotting patterns that grow by adding (arithmetic) or by multiplying (geometric), then using formulae to solve real problems, including money and growth. Work through each section in order and check your reasoning against the worked answers.

Revision summary

- Arithmetic sequence: constant difference d. nth term: $a_n = a + (n-1)d$. Sum: $S_n = n/2(2a + (n-1)d)$ or $n/2(a + l)$, where l is the last term.
- Geometric sequence: constant ratio r. nth term: $a_n = a \cdot r^{n-1}$. Sum: $S_n = a(r^n - 1)/(r - 1)$.
- Sum to infinity (only when $-1 < r < 1$): $S\infty = a/(1 - r)$.
- Sigma notation: Σ from k=1 to n of f(k) means add f(1) + f(2) + … + f(n). Read off the first term, the rule, and how many terms.
- Growth/decay: multiply by (1 + rate) each period → geometric. A quantity increasing $x each period → arithmetic.
- Compound interest amount after n periods: $A = P(1 + i)^n$. To solve for n, use logarithms: $n = \log(\text{target}/P) \div \log(1 + i)$.
- To decide arithmetic vs geometric: check if consecutive terms have a common difference (subtract) or a common ratio (divide).
- Solving S_n = a total often gives a quadratic in n, reject any negative or non-whole solution.

Key skills

1. An arithmetic sequence is 5, 8, 11, 14, … Find the 20th term.

2. Find the sum of the first 15 terms of the arithmetic series 4 + 7 + 10 + …

3. A geometric sequence is 3, 6, 12, 24, … Find the 8th term.

4. Evaluate Σ from k=1 to 5 of (2k + 1).

Achieved

5. A gym in Avondale charges $40 in the first month, then increases the monthly fee by $6 each month. How much is the fee in the 12th month?

6. A fish population in a lake starts at 500 and increases by 8% each year. Estimate the population after 6 years, to the nearest whole fish.

7. Evaluate Σ from k=1 to 10 of (3k − 2).

8. Mia saves $50 in her first week, then adds $5 more than the week before every week. How much has she saved in total after 12 weeks?

Merit

9. In an arithmetic sequence the 4th term is 23 and the 9th term is 48. Find the first term and common difference, then the sum of the first 20 terms.

10. A ball is dropped from a height of 2 m. Each bounce reaches 70% of the previous height. Find the total vertical distance the ball travels before coming to rest.

11. Ravi invests $2000 at 5% p.a. compound interest. Using logarithms, find the least whole number of years for the investment to first exceed $3000.

Excellence

12. Prove that the sum of the first n odd numbers, $1 + 3 + 5 + \ldots + (2n - 1)$, always equals n^2.

13. A geometric series has the sum of its first two terms equal to 12 and a sum to infinity of 18. Find the possible values of a and r, and justify that the series converges.

14. A concert hall has rows of seats where the first row has 10 seats and each row has 2 more seats than the row in front. The hall holds a total of 1170 seats. Find the number of rows, and explain why only one solution is valid.

NCEA Level 2 · AS 91259: Trig: Rules, Bearings & Radians

Name: Date:

Kia ora! This standard is about picking the right trig tool, sine rule, cosine rule, area formula, or radian measure, and using it to crack real problems, including bearings and 3D. Work through each section in order; the answers show the full working a marker wants to see.

Revision summary

- Sine rule: $a/\sin A = b/\sin B = c/\sin C$. Use when you have a side–opposite-angle pair (find a side or an angle).
- Cosine rule: $c^2 = a^2 + b^2 - 2ab\cdot\cos C$. Use for SAS (find a side) or SSS (rearrange: $\cos C = (a^2+b^2-c^2)/(2ab)$ to find an angle).
- Area of a triangle = $\frac{1}{2}\, ab\cdot\sin C$ (two sides and the included angle). Heron's: Area = $\sqrt{(s(s-a)(s-b)(s-c))}$, $s = (a+b+c)/2$.
- Bearings are measured clockwise from north, written as three digits (e.g. 072°). Sketch a north line at each point and find the interior angle before applying a rule.
- Radians: $180° = \pi$ rad. Convert: degrees $\times \pi/180$ = radians; radians $\times 180/\pi$ = degrees.
- Arc length $L = r\theta$ and sector area $A = \frac{1}{2}r^2\theta$, with θ in radians.
- Segment area = $\frac{1}{2}r^2(\theta - \sin\theta)$; chord length = $2r\cdot\sin(\theta/2)$, θ in radians.
- Ambiguous case (SSA): $\sin B = b\cdot\sin A/a$ may give two answers (B and 180°−B), check each keeps $A+B < 180°$.

Key skills

1. In triangle ABC, angle A = 40°, angle B = 65° and side a = 12 cm. Find side b.

2. Two sides of a triangle are 8 m and 11 m with an included angle of 52°. Find the third side.

3. A triangle has sides 9 cm and 14 cm with an included angle of 38°. Find its area.

4. A sector has radius 10 cm and central angle 1.2 radians. Find the arc length and the sector area.

Achieved

5. A tramper leaves the hut and walks 8 km on a bearing of 120°. How far south of the hut is she?

6. In triangle ABC, side a = 15 cm is opposite angle A, and side b = 10 cm is opposite angle B = 35°. Find angle A.

7. A triangular paddock has sides 7 m, 9 m and 12 m. Find the size of its largest angle.

8. An arc of a circle of radius 6 cm has length 15 cm. Find the central angle in radians, then in degrees.

Merit

9. A yacht sails 12 km on a bearing of 050°, then turns and sails 8 km on a bearing of 140°. Find its straight-line distance from the start, and the bearing it must steer to return directly to the start.

10. A triangular garden has sides 18 m, 22 m and 26 m. Find its area.

11. In triangle ABC, side a = 9 cm is opposite angle A = 40°, and side b = 12 cm is opposite angle B. Show that there are two possible triangles and find both values of B.

Excellence

12. From point A on flat ground the angle of elevation to the top of a flagpole is 28°. Walking 40 m directly toward the base to point B, the angle of elevation becomes 45°. Find the height of the flagpole, justifying your method.

13. Prove that the area of any triangle can be written as Area = $(a^2 \cdot \sin B \cdot \sin C)/(2 \cdot \sin A)$, where a, b, c are sides opposite angles A, B, C.

14. A circle has radius 8 cm. A chord of the circle is 10 cm long. Find the area of the minor segment cut off by the chord.

NCEA Level 2 · AS 91264: Making an Inference from Samples

Name: _____ Date: _____

This standard is all about taking samples from populations and using them to make a fair claim (an inference) about the wider group. Work through the sheet building box plots, calculating informal confidence intervals, and writing clear justified conclusions, that's what earns the marks.

Revision summary

- The statistical enquiry cycle (PPDAC): Problem, Plan, Data, Analysis, Conclusion. Your investigative question should compare a variable between two populations, e.g. 'I wonder if Year 12 boys at our school are taller than Year 12 girls, on average?'
- We take a SAMPLE to make an inference about the whole POPULATION (we don't have all the data). Use the median as the measure of centre for comparisons.
- Five-number summary: minimum, lower quartile (Q1), median (Q2), upper quartile (Q3), maximum. IQR = Q3 – Q1. Box plot: box from Q1 to Q3, line at median, whiskers to min and max.
- Informal confidence interval for a median: median ± 1.5 × (IQR ÷ √n). This estimates where the true population median likely sits.
- To make a call: if the two informal confidence intervals do NOT overlap, you can make the call back to the population that one median is likely bigger. If they overlap, you cannot make a call.
- Always describe features in context: centre (median), spread (IQR/range), shape (symmetry, skew), and any unusual/outlying values.
- A good conclusion answers the question, refers to the sample evidence, makes a call back to the population (or says you can't), and mentions sampling variability.
- Bigger sample sizes give narrower confidence intervals, so a more reliable inference.

Key skills

1. The reaction times (in seconds) of 9 students on a computer test are: 0.28, 0.31, 0.33, 0.35, 0.36, 0.39, 0.41, 0.44, 0.52. Find the five-number summary.

2. For a sample the lower quartile is 152 cm and the upper quartile is 168 cm. Calculate the interquartile range (IQR).

3. A sample of 25 kiwifruit has a median weight of 92 g and an IQR of 20 g. Calculate the informal confidence interval for the population median.

4. Explain the difference between a sample and a population, using the example of investigating the weekly screen time of Year 11 students at your school.

Achieved

5. A student investigates whether Year 13 students at Avondale College sleep more on weekends than on weeknights. Write a suitable investigative comparison question and identify the variable and the two groups being compared.

6. A sample of 16 avocados from a Kaitaia orchard has weights (g): 180, 190, 195, 200, 205, 210, 210, 215, 220, 225, 230, 235, 240, 250, 260, 280. Find the median, Q1 and Q3, then describe how you would draw the box plot.

7. A survey of 36 café flat whites finds a median price of $5.20 and an IQR of $0.90. Calculate the informal confidence interval for the median price and interpret it in context.

8. Two box plots show the heights of sampled netball players (median 172 cm) and basketball players (median 179 cm). The netball confidence interval is 168–176 cm and the basketball interval is 175–183 cm. State whether you can make a call about the populations and why.

Merit

9. A student compares reaction times (seconds) of a sample of 30 Year 12 students who play video games regularly (median 0.34, IQR 0.09) with 30 who do not (median 0.41, IQR 0.12). Calculate both informal confidence intervals, compare them, and make a call.

10. A sample of 50 commute times (minutes) to an Auckland CBD workplace has median 34, Q1 = 26, Q3 = 46, minimum 15, maximum 78. Describe the distribution referring to centre, spread and shape, and explain what the shape suggests about commute times.

11. Two students take separate samples of the same population of Year 9 running times. Sample A has n = 20 (median 62 s, IQR 10 s); Sample B has n = 80 (median 62 s, IQR 10 s). Calculate both confidence intervals and explain how sample size affects the reliability of the inference.

Excellence

12. A student concludes: 'Because the median height of my sample of boys (170 cm) is bigger than my sample of girls (163 cm), all boys at the school are taller than all girls.' Critique this conclusion and explain what a statistically valid conclusion should include.

13. Explain why the informal confidence interval uses median $\pm$ 1.5 $\times$ (IQR $\div$ $\sqrt{n}$), and why we compare intervals rather than simply comparing the two sample medians directly. Reason in terms of what would happen if the samples were taken again.

14. Two samples of tomato weights give overlapping confidence intervals, so a call cannot be made. A student says 'This proves the two populations have equal median weights.' Explain why this is incorrect, and describe two changes to the investigation that could allow a valid call to be made if a real difference exists.

NCEA Level 2 · AS 91269: Systems of Equations

Name: _____ Date: _____

This standard is all about solving two or three equations at once and using them to answer real questions. Work through each section in order, always writing your method clearly and checking your answer back in the original equations.

Revision summary

- A system of equations is two or more equations that share the same unknowns; a solution makes every equation true at once.
- Substitution: rearrange one equation for a variable, then substitute into the other. Best when a variable already has coefficient 1.
- Elimination: add or subtract multiples of the equations to cancel one variable. Best when coefficients line up neatly.
- Two linear equations in two unknowns usually give one solution (lines cross once). Parallel lines = no solution; identical lines = infinitely many solutions.
- For 3 unknowns: eliminate one variable using two different pairs of equations, giving a 2×2 system; solve that, then back-substitute.
- Modelling steps: define your variables clearly (with units), write one equation per condition, solve, then interpret the answer in context.
- Always check: substitute your solution back into the ORIGINAL equations and confirm the answer makes sense (e.g. no negative prices).
- Watch the units and the question, sometimes you need a combination (like total cost) rather than just x and y.

Key skills

1. Solve by substitution: $y = 2x + 1$ and $3x + y = 16$.

2. Solve by elimination: $2x + 3y = 12$ and $2x - y = 4$.

3. Solve: $4a + 5b = 23$ and $3a - 2b = 5$.

4. Check whether $x = 5$, $y = -2$ is the solution to $3x + 4y = 7$ and $x - 2y = 9$.

Achieved

5. At the Avondale market, 3 kererū-print tote bags and 2 keyrings cost $37. 1 tote bag and 4 keyrings cost $29. Find the price of one tote bag and one keyring.

6. A rectangle has perimeter 46 cm. Its length is 5 cm more than its width. Find the length and width.

7. Tickets to the Te Atatū school gala are $8 for adults and $5 for children. On Saturday 240 tickets were sold for a total of $1560. How many of each were sold?

8. Solve the system: $x + y = 10$, $y + z = 14$, $x + z = 12$.

Merit

9. A café in Kingsland sells three sizes of hot chocolate. A small plus a medium costs $10. A medium plus a large costs $13. A small plus a large costs $11. Find the price of each size and explain which size gives the best value per drink.

10. Two mobile plans: Plan A charges a $20 monthly fee plus $0.10 per text. Plan B charges a $35 monthly fee plus $0.05 per text. Write equations for the monthly cost of each and find the number of texts where both cost the same. Explain which plan is cheaper for someone sending 400 texts a month.

11. A number of $2 and $5 koha envelopes were collected at a fundraiser. There were 3 times as many $2 envelopes as $5 envelopes, and the total collected was $132. How many envelopes were there altogether?

Excellence

12. For what value of k does the system $2x + 3y = 12$ and $4x + ky = 20$ have no solution? Justify your answer, and state what happens for all other values of k.

13. A three-digit number has digits that add to 15. The tens digit is twice the hundreds digit. If the hundreds and units digits are swapped, the number decreases by 297. Find the original number and justify that it is unique.

14. Three friends compare weekly savings. Aroha saves $a, Ben saves $b, and Cara saves $c per week. Together they save $90 a week. Aroha saves $10 more than Ben. Cara saves the same as Aroha and Ben combined. Find each amount, then generalise: if the total is $T (with the same two relationships), express each person's saving in terms of T.

Answer key (for parents and tutors)

1. Simplify $(3a^2b)^3 \times 2ab^2$.
   $(3a^2b)^3 = 27a^6b^3$. Multiply by $2ab^2$: $27a^6b^3 \times 2ab^2 = 54a^7b^5$.
   Achieved: applies index laws to a product raised to a power.

2. Factorise $x^2 + 3x - 10$.
   Need two numbers with product -10 and sum $+3$: they are $+5$ and -2. So $x^2 + 3x - 10 = (x + 5)(x - 2)$.
   Achieved: factorises a monic quadratic.

3. Solve $2^x = 32$.
   $32 = 2^5$, so $2^x = 2^5$, therefore $x = 5$.
   Achieved: solves an exponential by matching bases.

4. Evaluate $\log_3 81$.
   $81 = 3^4$, so $\log_3 81 = 4$.
   Achieved: applies the definition of a logarithm.

5. Solve $x^2 - 5x + 6 = 0$ by factorising.
   Two numbers with product 6 and sum -5 are -2 and -3. So $(x - 2)(x - 3) = 0$, giving $x = 2$ or $x = 3$.
   Achieved: selects factorising to solve a quadratic.

6. Simplify $(12x^5y^3) \div (4x^2y^5)$.
   $12 \div 4 = 3$. $x^5 \div x^2 = x^3$. $y^3 \div y^5 = y^{-2} = 1/y^2$. So the expression $= 3x^3/y^2$.
   Achieved: applies index laws including a negative power.

7. Use the quadratic formula to solve $2x^2 + 3x - 4 = 0$. Give exact answers.
   $a = 2, b = 3, c = -4$. $\Delta = 3^2 - 4(2)(-4) = 9 + 32 = 41$. $x = (-3 \pm \sqrt{41}) \div (2 \times 2) = (-3 \pm \sqrt{41}) \div 4$. So $x = (-3 + \sqrt{41})/4$ or $x = (-3 - \sqrt{41})/4$.
   Achieved: applies the quadratic formula correctly to a non-factorising quadratic.

8. Solve $3^(x+1) = 81$.
   $81 = 3^4$, so $3^(x+1) = 3^4$, giving $x + 1 = 4$, therefore $x = 3$.
   Achieved: solves an exponential equation using equal bases.

9. A rectangular deck at a bach in Raglan has a length 3 m longer than its width. The area of the deck is 40 m². Form an equation and find the dimensions of the deck.
   Let width = x metres, so length = $x + 3$. Area: $x(x + 3) = 40$, which gives $x^2 + 3x - 40 = 0$. Factorise: $(x + 8)(x - 5) = 0$, so $x = -8$ or $x = 5$. A width cannot be negative, so $x = 5$. Width = 5 m, length = 5 + 3 = 8 m. Check: $5 \times 8 = 40$ m². ✓
   Merit: sets up a quadratic model from context and rejects the invalid solution.

10. Write $3/(x - 2) + 2/(x + 1)$ as a single simplified fraction.
    Common denominator is $(x - 2)(x + 1)$. $= [3(x + 1) + 2(x - 2)] \div [(x - 2)(x + 1)] = (3x + 3 + 2x - 4) \div [(x - 2)(x + 1)] = (5x - 1) \div [(x - 2)(x + 1)]$.
    Merit: links equivalent fractions through a common denominator in a multi-step manipulation.

11. Solve $9^x = 27^(x - 2)$.
    Write each side base 3: $9 = 3^2$ and $27 = 3^3$. So $(3^2)^x = (3^3)^(x - 2)$, giving $3^(2x) = 3^(3(x - 2))$. Equate powers: $2x = 3x - 6$, so $6 = x$. Therefore $x = 6$.
    Merit: connects two different bases to a common base to solve.

12. For what values of k does the equation $x^2 + kx + 4 = 0$ have two distinct real roots? Justify your answer.
    Two distinct real roots require the discriminant to be positive: $\Delta = b^2 - 4ac > 0$. Here $a = 1, b = k, c = 4$, so $\Delta = k^2 - 16 > 0$. Then $k^2 > 16$, which means $k < -4$ or $k > 4$. At $k = \pm 4$ there is exactly one repeated root, and for $-4 < k < 4$ there are no real roots. So two distinct real roots occur when $k < -4$ or $k > 4$.
    Excellence: uses the discriminant to reason about and justify a range of parameter values.

13. Solve $\log_2 x + \log_2(x - 2) = 3$, showing why any invalid solution must be rejected.
    Combine the logs: $\log_2[x(x - 2)] = 3$. Rewrite in index form: $x(x - 2) = 2^3 = 8$, so $x^2 - 2x - 8 = 0$. Factorise: $(x - 4)(x + 2) = 0$, giving $x = 4$ or $x = -2$. Check the domain: logs require positive arguments. If $x = -2$, then $\log_2 x = \log_2(-2)$ is

undefined, so $x = -2$ must be rejected. Only $x = 4$ works ($\log_2 4 + \log_2 2 = 2 + 1 = 3$ ✓).

Excellence: solves a log equation and justifies rejecting a solution using domain reasoning.

14. Prove that the difference between the squares of any two consecutive odd numbers is always a multiple of 8.

Let the two consecutive odd numbers be $2n - 1$ and $2n + 1$ (for integer n). Their squares differ by: $(2n + 1)^2 - (2n - 1)^2 = (4n^2 + 4n + 1) - (4n^2 - 4n + 1) = 8n$. Since n is an integer, $8n$ is a multiple of 8. Therefore the difference between the squares of any two consecutive odd numbers is always a multiple of 8.

Excellence: uses algebraic generalisation to construct a proof.

Answer key (for parents and tutors)

1. Differentiate $y = 4x^3 - 5x^2 + 7x - 2$.
   $dy/dx = 12x^2 - 10x + 7$. (Bring down the power and reduce it by one for each term; the constant -2 differentiates to 0.)
   Achieved: applies the power rule to a polynomial.

2. Find $\int (6x^2 + 4x - 3)\,dx$.
   $\int (6x^2 + 4x - 3)\,dx = 6x^3/3 + 4x^2/2 - 3x + c = 2x^3 + 2x^2 - 3x + c$. Remember the + c.
   Achieved: applies the integration rule and includes the constant.

3. Find the gradient of the curve $y = x^2 - 6x + 5$ at the point where $x = 4$.
   $dy/dx = 2x - 6$. At $x = 4$: gradient $= 2(4) - 6 = 2$.
   Achieved: evaluates the gradient function at a point.

4. A ball's height is $h = 20t - 5t^2$ metres. Find its velocity function.
   $v = dh/dt = 20 - 10t$ (metres per second).
   Achieved: differentiates displacement to obtain velocity.

5. Find the equation of the tangent to $y = x^2 - 3x + 1$ at the point where $x = 2$.
   When $x = 2$: $y = 4 - 6 + 1 = -1$, so the point is $(2, -1)$. $dy/dx = 2x - 3$, so at $x = 2$ the gradient $m = 2(2) - 3 = 1$. Tangent: $y - (-1) = 1(x - 2) \rightarrow y = x - 3$.
   Achieved: finds a point and gradient, then writes the tangent equation.

6. The gradient function of a curve is $dy/dx = 3x^2 - 4x$. The curve passes through $(1, 2)$. Find y as a function of x.
   $y = \int (3x^2 - 4x)\,dx = x^3 - 2x^2 + c$. Substitute $(1, 2)$: $2 = 1 - 2 + c \rightarrow c = 3$. So $y = x^3 - 2x^2 + 3$.
   Achieved: anti-differentiates and uses a point to find c.

7. Find the coordinates of the turning point of $y = x^2 - 8x + 10$.
   $dy/dx = 2x - 8 = 0 \rightarrow x = 4$. Then $y = 16 - 32 + 10 = -6$. Turning point at $(4, -6)$. (It is a minimum since the x^2 coefficient is positive.)
   Achieved: sets the derivative to zero to locate a turning point.

8. A drone rises so its height is $h = t^3 - 6t^2 + 9t$ metres, where t is in seconds. Find its velocity when $t = 1$ s.
   $v = dh/dt = 3t^2 - 12t + 9$. At $t = 1$: $v = 3 - 12 + 9 = 0$ m s^{-1} (the drone is momentarily at rest).
   Achieved: applies differentiation in a kinematics context.

9. A rectangular vegetable garden in Avondale is to be fenced against a straight wall, so only three sides need fencing. There are 60 m of fencing available. Find the dimensions that give the maximum area.
   Let the two sides perpendicular to the wall be x, and the side parallel be y. Fencing: $2x + y = 60$, so $y = 60 - 2x$. Area $A = xy = x(60 - 2x) = 60x - 2x^2$. $dA/dx = 60 - 4x = 0 \rightarrow x = 15$. Then $y = 60 - 30 = 30$. $d^2A/dx^2 = -4 < 0$, so this is a maximum. Dimensions: 15 m by 30 m, giving area = 450 m^2.
   Merit: builds a model, links constraint to area, optimises and verifies it is a maximum.

10. A curve has gradient function $dy/dx = 6x - 12$. Find the x-values where the curve is decreasing, and find where the curve has a turning point.
    The curve decreases where $dy/dx < 0$: $6x - 12 < 0 \rightarrow x < 2$. Turning point where $dy/dx = 0$: $6x - 12 = 0 \rightarrow x = 2$. So the curve decreases for $x < 2$, increases for $x > 2$, with a turning point at $x = 2$ (a minimum, since the gradient changes from negative to positive).
    Merit: links the sign of the derivative to increasing/decreasing behaviour and turning points.

11. A particle moves so its velocity is $v = 3t^2 - 12t + 9$ m s^{-1}. It starts at position $s = 5$ m when $t = 0$. Find its displacement function and the times when it is at rest.
    At rest when $v = 0$: $3t^2 - 12t + 9 = 0 \rightarrow t^2 - 4t + 3 = 0 \rightarrow (t - 1)(t - 3) = 0 \rightarrow t = 1$ s and $t = 3$ s. Displacement: $s = \int v\,dt = t^3 - 6t^2 + 9t + c$. At $t = 0$, $s = 5$, so $c = 5$. Thus $s = t^3 - 6t^2 + 9t + 5$.
    Merit: connects integration with initial conditions and solves for rest points.

12. The tangent to the curve $y = x^2 + kx - 4$ at the point where $x = 1$ has gradient 5. Find k, then find the equation of that tangent.

$dy/dx = 2x + k$. At $x = 1$: $2 + k = 5 \rightarrow k = 3$. So $y = x^2 + 3x - 4$. At $x = 1$: $y = 1 + 3 - 4 = 0$, point $(1, 0)$. Tangent: $y - 0 = 5(x - 1) \rightarrow y = 5x - 5$.

Merit: uses a gradient condition to find an unknown parameter, then constructs the tangent.

13. A cylindrical rainwater tank (no lid) is to hold a fixed volume V. Show that the surface area of metal used is least when the height equals the radius.

    Let radius r, height h. Volume: $V = \pi r^2 h$, so $h = V/(\pi r^2)$. Surface area (base + curved side) $S = \pi r^2 + 2\pi rh = \pi r^2 + 2\pi r(V/(\pi r^2)) = \pi r^2 + 2V/r$. $dS/dr = 2\pi r - 2V/r^2$. Set to 0: $2\pi r = 2V/r^2 \rightarrow \pi r^3 = V \rightarrow r^3 = V/\pi$. Since $V = \pi r^2 h$, $r^3 = r^2 h$, so $r = h$. $d^2S/dr^2 = 2\pi + 4V/r^3 > 0$, confirming a minimum. Therefore the surface area is least when $h = r$.

    Excellence: generalises with a symbolic volume and proves the optimal condition.

14. A curve $y = ax^3 + bx^2$ has a turning point at $(2, -4)$. Find a and b, justifying your reasoning.

    $dy/dx = 3ax^2 + 2bx$. At the turning point $x = 2$: $3a(4) + 2b(2) = 0 \rightarrow 12a + 4b = 0 \rightarrow b = -3a$. The point lies on the curve: $-4 = a(8) + b(4) = 8a + 4b$. Substitute $b = -3a$: $-4 = 8a + 4(-3a) = 8a - 12a = -4a \rightarrow a = 1$, so $b = -3$. Curve: $y = x^3 - 3x^2$. Check: $dy/dx = 3x^2 - 6x = 3x(x - 2)$, zero at $x = 2$ ✓, and $y = 8 - 12 = -4$ ✓.

    Excellence: sets up and solves simultaneous conditions from an unfamiliar constraint, with justification.

15. A ball is thrown upward with height $h = ut - 5t^2$ metres (u is the initial speed in m s^{-1}). Prove that the maximum height reached is $u^2/20$ metres, regardless of u.

    Velocity $v = dh/dt = u - 10t$. Maximum height when $v = 0$: $u - 10t = 0 \rightarrow t = u/10$. Substitute into h: $h = u(u/10) - 5(u/10)^2 = u^2/10 - 5(u^2/100) = u^2/10 - u^2/20 = 2u^2/20 - u^2/20 = u^2/20$. Since $d^2h/dt^2 = -10 < 0$, this is a maximum. Therefore the maximum height is $u^2/20$ for any u.

    Excellence: generalises the result algebraically and proves it holds for all values of the parameter.

Answer key (for parents and tutors)

1. A bag holds 5 red, 3 blue and 2 green marbles. One marble is drawn at random. Find P(blue).
   Total marbles = 5 + 3 + 2 = 10. P(blue) = 3 ÷ 10 = 0.3.
   Achieved: applies basic probability as favourable ÷ total.

2. In a survey of 200 people, 120 own a bike and of those 45 also own a scooter. Find P(owns a scooter given owns a bike).
   P(scooter given bike) = 45 ÷ 120 = 0.375.
   Achieved: applies the conditional probability method from a subgroup.

3. A spinner lands on 'win' with probability 0.4. It is spun twice. Draw/consider a tree and find P(win then win).
   Branches are independent, so multiply along the path: P(win, win) = 0.4 × 0.4 = 0.16.
   Achieved: multiplies along tree branches for independent events.

4. Heights are normally distributed with mean μ = 170 cm and standard deviation σ = 8 cm. Find the z-score for a height of 182 cm.
   $z = (x - \mu) \div \sigma = (182 - 170) \div 8 = 12 \div 8 = 1.5$.
   Achieved: standardises a value using the z-score formula.

5. At Avondale College, 300 students were asked if they walk to school. The two-way table shows: Junior students – 90 walk, 60 do not; Senior students – 50 walk, 100 do not. Find the probability a randomly chosen student walks to school.
   Total who walk = 90 + 50 = 140. Total students = 300. P(walks) = 140 ÷ 300 = 0.467 (3 s.f.).
   Achieved: reads a two-way table and computes a marginal probability.

6. A café finds 70% of customers order coffee. Of those who order coffee, 30% also order a muffin. Draw a tree for the first branch and find P(orders coffee and a muffin).
   P(coffee) = 0.7, P(muffin given coffee) = 0.3. Multiply along branches: P(coffee and muffin) = 0.7 × 0.3 = 0.21.
   Achieved: applies the multiplication rule along a tree path.

7. The time to fill a petrol tank is normally distributed with mean 4.5 minutes and standard deviation 0.8 minutes. Find the probability a fill takes less than 5 minutes.
   $z = (5 - 4.5) \div 0.8 = 0.625$. $P(z < 0.625) \approx 0.734$. So about 0.734 (73.4%).
   Achieved: standardises and reads a normal-distribution probability.

8. In a town, 40 of 500 dog owners reported their dog had fleas, while 12 of 300 cat owners reported fleas. Calculate the risk of fleas for dog owners and for cat owners.
   Risk (dogs) = 40 ÷ 500 = 0.08. Risk (cats) = 12 ÷ 300 = 0.04.
   Achieved: calculates risk as a relative frequency for two groups.

9. A factory has two machines. Machine A makes 60% of items with a 3% defect rate; Machine B makes 40% with a 5% defect rate. An item is chosen at random and found to be defective. Find the probability it came from Machine A.
   P(A and defective) = 0.60 × 0.03 = 0.018. P(B and defective) = 0.40 × 0.05 = 0.020. P(defective) = 0.018 + 0.020 = 0.038. P(A given defective) = 0.018 ÷ 0.038 = 0.474 (3 s.f.).
   Merit: links a tree diagram to conditional probability (reverse conditioning).

10. Using the flea data (dogs: risk 0.08, cats: risk 0.04), calculate the relative risk of fleas for dogs compared with cats and interpret it in context.
    Relative risk = risk(dogs) ÷ risk(cats) = 0.08 ÷ 0.04 = 2. Interpretation: dog owners in this town are twice as likely to report their pet having fleas as cat owners.
    Merit: computes relative risk and interprets it in context.

11. Bags of flour are filled to a mean of 1500 g with standard deviation 12 g, normally distributed. Bags under 1485 g are rejected. In a batch of 800 bags, estimate how many are rejected.
    $z = (1485 - 1500) \div 12 = -1.25$. $P(x < 1485) = P(z < -1.25) \approx 0.1056$. Expected rejects = 0.1056 × 800 ≈ 84 bags.
    Merit: links a normal probability to an expected count for a real batch.

12. A medical test for a condition is 96% accurate for people who have it (true positive) and gives a false positive 5% of the time. In the population, 2% of people have the condition. A person tests positive. Find the probability they actually have the condition, and comment on why the result may seem surprising.
P(have) = 0.02, P(positive given have) = 0.96, so P(have and positive) = 0.02 × 0.96 = 0.0192. P(not have) = 0.98, P(positive given not have) = 0.05, so P(not have and positive) = 0.98 × 0.05 = 0.049. P(positive) = 0.0192 + 0.049 = 0.0682. P(have given positive) = 0.0192 ÷ 0.0682 = 0.282 (3 s.f.). Comment: even with a fairly accurate test, only about 28% of positives truly have the condition, because the condition is rare, so the many false positives from the large healthy group outnumber the true positives.
Excellence: justifies a counter-intuitive result by reasoning about base rates.

13. Two events A and B satisfy P(A) = 0.5, P(B) = 0.4 and P(A and B) = 0.2. Determine whether A and B are independent, and prove your conclusion using probability rules.
For independence we require P(A and B) = P(A) × P(B). Compute P(A) × P(B) = 0.5 × 0.4 = 0.2, which equals P(A and B) = 0.2. Since they are equal, A and B are independent. Equivalently, P(A given B) = P(A and B) ÷ P(B) = 0.2 ÷ 0.4 = 0.5 = P(A), confirming B does not change the probability of A.
Excellence: justifies independence formally using two equivalent criteria.

14. The lifetimes of two brands of LED bulb are normally distributed. Brand X: mean 20000 h, σ = 2000 h. Brand Y: mean 22000 h, σ = 3500 h. A retailer wants bulbs lasting at least 18000 h. Which brand gives a higher chance of meeting this, and by how much? Justify whether a larger mean always guarantees a better result here.
Brand X: z = (18000 − 20000) ÷ 2000 = −1.0, P(x ≥ 18000) = P(z ≥ −1.0) ≈ 0.8413. Brand Y: z = (18000 − 22000) ÷ 3500 = −1.143, P(x ≥ 18000) = P(z ≥ −1.143) ≈ 0.8735. Brand Y is better here, by about 0.8735 − 0.8413 = 0.032 (about 3 percentage points). Justification: a larger mean does not automatically guarantee a better chance at a threshold, because the larger spread (σ) of Brand Y works against it. In this case Y's higher mean outweighs its larger σ at the 18000 h cut-off, but at a higher threshold the greater spread could reverse the advantage, the outcome depends on both mean and standard deviation relative to the threshold.
Excellence: generalises about the combined effect of mean and spread with justification.

Answer key (for parents and tutors)

1. Find the distance between A(1, 2) and B(7, 10).
   $d = \sqrt{((7-1)^2 + (10-2)^2)} = \sqrt{(6^2 + 8^2)} = \sqrt{(36 + 64)} = \sqrt{100} = 10$.
   Achieved: applies the distance formula.

2. Find the midpoint of P(−3, 5) and Q(7, −1).
   $M = ((-3+7)/2 , (5+(-1))/2) = (4/2 , 4/2) = (2, 2)$.
   Achieved: applies the midpoint formula.

3. Find the gradient of the line through C(2, 3) and D(6, 11).
   $m = (11 - 3)/(6 - 2) = 8/4 = 2$.
   Achieved: applies the gradient formula.

4. Write the equation of the line with gradient 3 that passes through (2, −1).
   $y - (-1) = 3(x - 2) \rightarrow y + 1 = 3x - 6 \rightarrow y = 3x - 7$.
   Achieved: forms a line equation from a point and gradient.

5. Find the equation of the straight line passing through the points (1, 2) and (4, 11).
   Gradient $m = (11 - 2)/(4 - 1) = 9/3 = 3$. Using (1, 2): $y - 2 = 3(x - 1) \rightarrow y = 3x - 3 + 2 \rightarrow y = 3x - 1$.
   Achieved: selects gradient then forms the line equation.

6. A line has equation $y = 2x + 5$. Find the equation of the line perpendicular to it that passes through (4, 1).
   Gradient of given line = 2, so perpendicular gradient = $-1/2$. Using (4, 1): $y - 1 = -1/2(x - 4) \rightarrow y - 1 = -1/2\,x + 2 \rightarrow y = -1/2\,x + 3$.
   Achieved: applies the perpendicular gradient rule.

7. Find the equation of the line parallel to $y = -3x + 2$ that passes through (2, 5).
   Parallel lines have equal gradients, so $m = -3$. Using (2, 5): $y - 5 = -3(x - 2) \rightarrow y - 5 = -3x + 6 \rightarrow y = -3x + 11$.
   Achieved: applies the parallel gradient rule.

8. On a walking-track map (units in km), a café is at (2, 3) and a lookout is at (8, 11). How far apart are they in a straight line?
   $d = \sqrt{((8-2)^2 + (11-3)^2)} = \sqrt{(6^2 + 8^2)} = \sqrt{(36 + 64)} = \sqrt{100} = 10$ km.
   Achieved: applies distance formula in a modelling context.

9. A triangle has vertices A(−2, 1), B(4, 3) and C(2, 9). Show that the triangle has a right angle, and state where it is.
   Gradient $AB = (3 - 1)/(4 - (-2)) = 2/6 = 1/3$. Gradient $BC = (9 - 3)/(2 - 4) = 6/(-2) = -3$. Product of gradients = $(1/3) \times (-3) = -1$, so AB is perpendicular to BC. Therefore the right angle is at B.
   Merit: links gradients of two sides to prove perpendicularity.

10. Two straight roads are modelled by $y = 2x - 3$ and $x + y = 9$. Find the coordinates of the point where the roads cross.
    Substitute $y = 2x - 3$ into $x + y = 9$: $x + (2x - 3) = 9 \rightarrow 3x - 3 = 9 \rightarrow 3x = 12 \rightarrow x = 4$. Then $y = 2(4) - 3 = 5$. The roads cross at (4, 5).
    Merit: connects two representations by solving simultaneously.

11. Find the equation of the perpendicular bisector of the line segment joining A(1, 2) and B(5, 8).
    Midpoint of $AB = ((1+5)/2 , (2+8)/2) = (3, 5)$. Gradient $AB = (8 - 2)/(5 - 1) = 6/4 = 3/2$, so perpendicular gradient = $-2/3$. Through (3, 5): $y - 5 = -2/3(x - 3) \rightarrow y - 5 = -2/3\,x + 2 \rightarrow y = -2/3\,x + 7$.
    Merit: combines midpoint and perpendicular gradient in a multi-step solution.

12. A quadrilateral has vertices A(1, 1), B(5, 3), C(7, 7) and D(3, 5). Prove that ABCD is a rhombus.
    Gradients: $AB = (3-1)/(5-1) = 2/4 = 1/2$; $DC = (7-5)/(7-3) = 2/4 = 1/2$, so $AB \parallel DC$. $AD = (5-1)/(3-1) = 4/2 = 2$; $BC = (7-3)/(7-5) = 4/2 = 2$, so $AD \parallel BC$. Both pairs of opposite sides are parallel, so ABCD is a parallelogram. Side lengths: $AB = \sqrt{(4^2 + 2^2)} = \sqrt{20}$; $AD = \sqrt{(2^2 + 4^2)} = \sqrt{20}$. Adjacent sides are equal, and since a parallelogram with two equal adjacent sides has all four sides equal (AB = DC and AD = BC by the parallelogram property), all sides = $\sqrt{20}$. A parallelogram with all sides equal is a rhombus. ■
    Excellence: constructs a full logical proof combining gradients and lengths.

13. A stream through a park follows the line $y = 2x$ (the park entrance is the origin). A pōhutukawa tree stands at T(5, 0). Find the shortest distance from the tree to the stream, and the coordinates of the nearest point on the stream. Justify

why this is the shortest distance.

The shortest distance from a point to a line is along the perpendicular, because any other path forms the hypotenuse of a right triangle and is therefore longer. Stream gradient = 2, so the perpendicular gradient = −1/2. Line from T(5, 0): $y - 0 = -1/2(x - 5) \rightarrow y = -1/2\,x + 5/2$. Intersect with $y = 2x$: $2x = -1/2\,x + 5/2 \rightarrow 2.5x = 2.5 \rightarrow x = 1$, $y = 2$. Nearest point N = (1, 2). Distance TN = $\sqrt{((5-1)^2 + (0-2)^2)} = \sqrt{(16 + 4)} = \sqrt{20} = 2\sqrt{5}$ (≈ 4.47 units).

Excellence: solves an unfamiliar problem and justifies the perpendicular gives the minimum.

14. Two lines are given by $y = (k - 1)x + 3$ and $2x + 4y = 7$, where k is a constant. Find the value of k that makes the lines perpendicular, and explain your reasoning.

Rearrange the second line: $4y = -2x + 7 \rightarrow y = -1/2\,x + 7/4$, so its gradient is −1/2. For perpendicular lines the gradients multiply to −1: $(k - 1) \times (-1/2) = -1$. Multiply both sides by −2: $k - 1 = 2$, so k = 3. Reasoning: only when the first gradient (k − 1) is the negative reciprocal of −1/2, i.e. 2, are the lines perpendicular; this occurs at k = 3.

Excellence: generalises the perpendicular condition with an algebraic parameter and justifies it.

Answer key (for parents and tutors)

1. State the gradient and y-intercept of the line $y = 3x - 5$, then find the x-intercept.
   Gradient $m = 3$, y-intercept $c = -5$. For the x-intercept set $y = 0$: $0 = 3x - 5$, so $3x = 5$, $x = 5/3 \approx 1.67$. x-intercept is $(5/3, 0)$.
   Achieved: reads features directly from $y = mx + c$ form.

2. Write the coordinates of the vertex of $y = (x - 4)^2 + 2$ and state whether it is a maximum or minimum.
   The form $y = a(x - h)^2 + k$ gives vertex $(h, k) = (4, 2)$. Since $a = 1 > 0$ the parabola opens upward, so the vertex is a minimum.
   Achieved: identifies vertex and orientation from completed-square form.

3. Describe the transformation that takes $y = x^2$ to $y = x^2 - 6$.
   Adding -6 to the function shifts every point down by 6 units. It is a vertical translation of 6 units downward.
   Key skill: names a vertical translation.

4. For the exponential $y = 2^x$, state the y-intercept and the equation of the horizontal asymptote.
   At $x = 0$, $y = 2^0 = 1$, so the y-intercept is $(0, 1)$. As $x \to -\infty$, $y \to 0$, so the horizontal asymptote is $y = 0$.
   Key skill: reads intercept and asymptote of a basic exponential.

5. A water tank at a bach near Muriwai starts with 500 L and is filled at a constant 40 L per minute. Write an equation for the volume V after t minutes and find the volume after 12 minutes.
   Constant rate means a linear model: $V = 500 + 40t$. After 12 minutes: $V = 500 + 40 \times 12 = 500 + 480 = 980$ L.
   Achieved: selects a linear model and applies it.

6. Sketch $y = (x + 1)(x - 5)$ by finding its x-intercepts, axis of symmetry and vertex.
   x-intercepts where $y = 0$: $x = -1$ and $x = 5$, i.e. points $(-1, 0)$ and $(5, 0)$. Axis of symmetry is midway: $x = (-1 + 5)/2 = 2$. Vertex y-value: $y = (2 + 1)(2 - 5) = 3 \times (-3) = -9$, so vertex $(2, -9)$. Parabola opens upward ($a = 1$) through the two intercepts with minimum at $(2, -9)$.
   Achieved: finds roots, axis and vertex to sketch a parabola.

7. The population of a rabbit colony is modelled by $P = 30 \times 1.5^t$, where t is in years. Find the starting population and the population after 3 years.
   Starting population at $t = 0$: $P = 30 \times 1.5^0 = 30 \times 1 = 30$ rabbits. After 3 years: $P = 30 \times 1.5^3 = 30 \times 3.375 = 101.25$, so about 101 rabbits.
   Achieved: applies an exponential model at given values.

8. A line passes through $(0, 4)$ and $(2, 10)$. Find its equation in the form $y = mx + c$.
   Gradient $m = (10 - 4)/(2 - 0) = 6/2 = 3$. The y-intercept is 4 (point at $x = 0$). So $y = 3x + 4$.
   Achieved: forms a linear equation from two points.

9. A drone is launched from a 5 m platform. Its height (m) after t seconds is $h = -5t^2 + 20t + 5$. Find the maximum height reached and when it hits the ground.
   Maximum at the vertex. Axis of symmetry: $t = -b/(2a) = -20/(2 \times -5) = 2$ s. Max height: $h = -5(2)^2 + 20(2) + 5 = -20 + 40 + 5 = 25$ m. Hits ground when $h = 0$: $-5t^2 + 20t + 5 = 0$, divide by -5: $t^2 - 4t - 1 = 0$, $t = (4 \pm \sqrt{(16 + 4)})/2 = (4 \pm \sqrt{20})/2 = 2 \pm \sqrt{5}$. Taking the positive root: $t = 2 + 2.236 \approx 4.24$ s.
   Merit: links vertex and roots to a physical situation via multi-step work.

10. The graph of $y = x^2$ is transformed to $y = 2(x - 3)^2 - 4$. Describe the sequence of transformations, then state the coordinates of the new vertex and the y-intercept.
    From $y = x^2$: horizontal shift right 3 (from $x - 3$), vertical stretch by factor 2 (the $\times 2$), then vertical shift down 4 (the $- 4$). New vertex is $(3, -4)$. y-intercept: set $x = 0$: $y = 2(0 - 3)^2 - 4 = 2 \times 9 - 4 = 18 - 4 = 14$, so $(0, 14)$.
    Merit: connects multiple transformations and derives resulting features.

11. Two courier companies charge for a delivery. FastVan charges $8 plus $2 per km; QuickGo charges a flat $20. Write an equation for each, find the distance where the costs are equal, and state which is cheaper for a 10 km trip.
    FastVan: $C = 8 + 2d$. QuickGo: $C = 20$. Equal when $8 + 2d = 20$, so $2d = 12$, $d = 6$ km. For 10 km FastVan costs $8 + 2 \times$

10 = $28, which is more than QuickGo's $20, so QuickGo is cheaper for 10 km (FastVan is cheaper below 6 km).
Merit: connects two representations to compare and interpret an intersection.

12. A parabola has x-intercepts at $x = -2$ and $x = 6$ and passes through $(2, -32)$. Find its equation in the form $y = ax^2 + bx + c$, and justify why the vertex must lie on $x = 2$.
With roots -2 and 6: $y = a(x + 2)(x - 6)$. Substitute $(2, -32)$: $-32 = a(2 + 2)(2 - 6) = a(4)(-4) = -16a$, so $a = 2$. Then $y = 2(x + 2)(x - 6) = 2(x^2 - 4x - 12) = 2x^2 - 8x - 24$. The axis of symmetry is midway between the roots: $x = (-2 + 6)/2 = 2$, so the vertex lies on $x = 2$, which is also why the given point $(2, -32)$ is exactly the vertex.
Excellence: reconstructs and justifies a model from features, recognising the point as the vertex.

13. An exponential model $y = a \cdot b^x$ passes through $(0, 12)$ and $(2, 3)$. Find a and b, and explain what value of b tells you about the situation and the behaviour of the graph.
At $x = 0$: $y = a \cdot b^0 = a = 12$. At $x = 2$: $3 = 12 \cdot b^2$, so $b^2 = 3/12 = 1/4$, $b = 1/2$ (taking the positive root). Model: $y = 12 \times (1/2)^x$. Since $0 < b < 1$ the quantity is decaying, it halves for every increase of 1 in x. The graph decreases toward the horizontal asymptote $y = 0$ but never reaches it, so y stays positive.
Excellence: solves for parameters and generalises the meaning of the base.

14. Prove that the line $y = x + k$ is tangent to the parabola $y = x^2$ only when $k = -1/4$, and state the point of tangency.
At intersection: $x^2 = x + k$, so $x^2 - x - k = 0$. A tangent means exactly one solution, i.e. the discriminant is zero: $b^2 - 4ac = (-1)^2 - 4(1)(-k) = 1 + 4k = 0$, giving $k = -1/4$. Then $x^2 - x + 1/4 = 0$, $(x - 1/2)^2 = 0$, so $x = 1/2$ and $y = (1/2)^2 = 1/4$. Point of tangency is $(1/2, 1/4)$.
Excellence: uses the discriminant condition to prove tangency and generalise.

Answer key (for parents and tutors)

1. An arithmetic sequence is 5, 8, 11, 14, … Find the 20th term.
   $a = 5$, $d = 3$. $a_{20} = a + 19d = 5 + 19\times3 = 5 + 57 = 62$.
   Achieved fluency: substitutes into $a_n = a + (n-1)d$.

2. Find the sum of the first 15 terms of the arithmetic series $4 + 7 + 10 + \ldots$
   $a = 4$, $d = 3$, $n = 15$. $S_{15} = 15/2(2\times4 + 14\times3) = 15/2(8 + 42) = 15/2 \times 50 = 375$.
   Achieved fluency: applies the arithmetic sum formula.

3. A geometric sequence is 3, 6, 12, 24, … Find the 8th term.
   $a = 3$, $r = 2$. $a_8 = a\cdot r^7 = 3 \times 2^7 = 3 \times 128 = 384$.
   Achieved fluency: applies $a_n = a\cdot r^{n-1}$.

4. Evaluate Σ from k=1 to 5 of $(2k + 1)$.
   Terms: k=1→3, k=2→5, k=3→7, k=4→9, k=5→11. Sum = 3 + 5 + 7 + 9 + 11 = 35.
   Achieved fluency: expands and sums sigma notation.

5. A gym in Avondale charges $40 in the first month, then increases the monthly fee by $6 each month. How much is the fee in the 12th month?
   Arithmetic: $a = 40$, $d = 6$. $a_{12} = 40 + 11\times6 = 40 + 66 = \$106$.
   Achieved: selects and applies $a_n = a + (n-1)d$ in context.

6. A fish population in a lake starts at 500 and increases by 8% each year. Estimate the population after 6 years, to the nearest whole fish.
   Geometric growth: multiply by 1.08 each year. After 6 years: $500 \times 1.08^6 = 500 \times 1.586874\ldots = 793.4 \approx 793$ fish.
   Achieved: applies compound growth with the multiplier 1.08.

7. Evaluate Σ from k=1 to 10 of $(3k - 2)$.
   This is arithmetic: first term (k=1) = 1, last term (k=10) = 28, n = 10. Sum = 10/2(1 + 28) = 5 × 29 = 145.
   Achieved: recognises sigma as an arithmetic series and sums it.

8. Mia saves $50 in her first week, then adds $5 more than the week before every week. How much has she saved in total after 12 weeks?
   Arithmetic: $a = 50$, $d = 5$, $n = 12$. $S_{12} = 12/2(2\times50 + 11\times5) = 6(100 + 55) = 6 \times 155 = \$930$.
   Achieved: applies the arithmetic sum formula to a savings context.

9. In an arithmetic sequence the 4th term is 23 and the 9th term is 48. Find the first term and common difference, then the sum of the first 20 terms.
   $d = (48 - 23)/(9 - 4) = 25/5 = 5$. $a = a_4 - 3d = 23 - 15 = 8$. $S_{20} = 20/2(2\times8 + 19\times5) = 10(16 + 95) = 10 \times 111 = 1110$.
   Merit: links two term equations to find a and d, then chains into the sum.

10. A ball is dropped from a height of 2 m. Each bounce reaches 70% of the previous height. Find the total vertical distance the ball travels before coming to rest.
    Initial drop = 2 m. First bounce up = $2\times0.7 = 1.4$ m, and the up-heights form a geometric series with $a = 1.4$, $r = 0.7$. Since $-1 < r < 1$: $S_\infty = 1.4/(1 - 0.7) = 1.4/0.3 = 4.667$ m. Each bounce goes up then down, so bounce distance = $2 \times 4.667 = 9.333$ m. Total = $2 + 9.333 = 11.33$ m (2 dp).
    Merit: links geometric sum to infinity with the up-and-down structure of the motion.

11. Ravi invests $2000 at 5% p.a. compound interest. Using logarithms, find the least whole number of years for the investment to first exceed $3000.
    Need $2000 \times 1.05^n > 3000 \rightarrow 1.05^n > 1.5$. Take logs: $n > \log(1.5)/\log(1.05) = 0.17609/0.02119 = 8.31$. So n = 9 years.
    Merit: connects compound growth to logarithms and interprets the answer as a whole year.

12. Prove that the sum of the first n odd numbers, $1 + 3 + 5 + \ldots + (2n - 1)$, always equals n^2.
    The odd numbers form an arithmetic series with $a = 1$, $d = 2$, and nth term $2n - 1$. Sum = $n/2(a + l) = n/2(1 + (2n - 1)) = n/2 \times 2n = n^2$. Since this holds for the general n using the arithmetic sum formula, the sum of the first n odd numbers is exactly n^2 for every positive integer n.
    Excellence: generalises and justifies a result algebraically for all n.

13. A geometric series has the sum of its first two terms equal to 12 and a sum to infinity of 18. Find the possible values of a and r, and justify that the series converges.
$S_\infty = a/(1 - r) = 18$, so $a = 18(1 - r)$. First two terms: $a + ar = a(1 + r) = 12$. Substitute: $18(1 - r)(1 + r) = 12 \rightarrow 18(1 - r^2) = 12 \rightarrow 1 - r^2 = 2/3 \rightarrow r^2 = 1/3 \rightarrow r = \pm 1/\sqrt{3} \approx \pm 0.577$. Both satisfy $|r| < 1$, so the series converges (each term shrinks in size). If $r = 0.577$, $a = 18(1 - 0.577) = 7.61$; if $r = -0.577$, $a = 18(1 + 0.577) = 28.39$.
Excellence: solves a two-condition system, finds both cases and justifies convergence.

14. A concert hall has rows of seats where the first row has 10 seats and each row has 2 more seats than the row in front. The hall holds a total of 1170 seats. Find the number of rows, and explain why only one solution is valid.
Arithmetic: $a = 10$, $d = 2$. $S_n = n/2(2 \times 10 + (n - 1)2) = n/2(20 + 2n - 2) = n/2(18 + 2n) = n(9 + n) = n^2 + 9n$. Set equal to 1170: $n^2 + 9n - 1170 = 0$. Discriminant $= 81 + 4680 = 4761 = 69^2$. $n = (-9 \pm 69)/2$, giving $n = 30$ or $n = -39$. The number of rows must be a positive whole number, so $n = -39$ is rejected. There are 30 rows.
Excellence: forms and solves a quadratic from a sum, and justifies rejecting the invalid root.

Answer key (for parents and tutors)

1. In triangle ABC, angle A = 40°, angle B = 65° and side a = 12 cm. Find side b.
   Sine rule: b/sinB = a/sinA, so b = a·sinB/sinA = 12·sin65°/sin40° = 12·0.9063/0.6428 = 16.9 cm.
   Achieved: selects and applies the sine rule to find a side.

2. Two sides of a triangle are 8 m and 11 m with an included angle of 52°. Find the third side.
   Cosine rule: $c^2 = 8^2 + 11^2 - 2 \cdot 8 \cdot 11 \cdot \cos 52° = 64 + 121 - 176 \cdot 0.6157 = 185 - 108.4 = 76.64$, so $c = \sqrt{76.64} = 8.75$ m.
   Achieved: applies the cosine rule (SAS) for a side.

3. A triangle has sides 9 cm and 14 cm with an included angle of 38°. Find its area.
   Area = $\frac{1}{2} \cdot 9 \cdot 14 \cdot \sin 38° = 63 \cdot 0.6157 = 38.8$ cm².
   Achieved: applies the ½ab·sinC area formula.

4. A sector has radius 10 cm and central angle 1.2 radians. Find the arc length and the sector area.
   Arc $L = r\theta = 10 \cdot 1.2 = 12$ cm. Sector area = $\frac{1}{2}r^2\theta = \frac{1}{2} \cdot 10^2 \cdot 1.2 = 60$ cm².
   Achieved: applies arc length and sector area formulae in radians.

5. A tramper leaves the hut and walks 8 km on a bearing of 120°. How far south of the hut is she?
   North component = $8 \cdot \cos 120° = 8 \cdot (-0.5) = -4$, i.e. 4 km. The negative shows the movement is southward, so she is 4 km south of the hut.
   Achieved: resolves a bearing into a directional component.

6. In triangle ABC, side a = 15 cm is opposite angle A, and side b = 10 cm is opposite angle B = 35°. Find angle A.
   Sine rule: sinA/a = sinB/b, so $\sin A = a \cdot \sin B / b = 15 \cdot \sin 35° / 10 = 15 \cdot 0.5736 / 10 = 0.8604$. $A = \sin^{-1}(0.8604) = 59.4°$.
   Achieved: applies the sine rule to find an angle.

7. A triangular paddock has sides 7 m, 9 m and 12 m. Find the size of its largest angle.
   The largest angle is opposite the longest side (12 m). $\cos C = (7^2 + 9^2 - 12^2)/(2 \cdot 7 \cdot 9) = (49 + 81 - 144)/126 = -14/126 = -0.1111$. $C = \cos^{-1}(-0.1111) = 96.4°$.
   Achieved: applies the cosine rule (SSS) to find an angle.

8. An arc of a circle of radius 6 cm has length 15 cm. Find the central angle in radians, then in degrees.
   $L = r\theta$, so $\theta = L/r = 15/6 = 2.5$ radians. In degrees: $2.5 \cdot 180/\pi = 143.2°$.
   Achieved: uses arc length formula and converts radians to degrees.

9. A yacht sails 12 km on a bearing of 050°, then turns and sails 8 km on a bearing of 140°. Find its straight-line distance from the start, and the bearing it must steer to return directly to the start.
   Break into east/north components. Leg 1: E = 12sin50° = 9.192, N = 12cos50° = 7.713. Leg 2: E = 8sin140° = 5.142, N = 8cos140° = −6.128. Totals: E = 14.334, N = 1.585. Distance = $\sqrt{(14.334^2 + 1.585^2)} = \sqrt{(205.46 + 2.51)} = \sqrt{207.97} = 14.4$ km. Bearing from start to yacht = $\tan^{-1}(E/N) = \tan^{-1}(14.334/1.585) = 83.7°$, i.e. 084°. To return, steer the reverse bearing: 084° + 180° = 264°.
   Merit: links two-leg bearings via components and reverses the bearing to solve a multi-step problem.

10. A triangular garden has sides 18 m, 22 m and 26 m. Find its area.
    Find the angle between the 18 m and 22 m sides (opposite 26 m): $\cos C = (18^2 + 22^2 - 26^2)/(2 \cdot 18 \cdot 22) = (324 + 484 - 676)/792 = 132/792 = 0.1667$, so $C = 80.4°$. Area = $\frac{1}{2} \cdot 18 \cdot 22 \cdot \sin 80.4° = 198 \cdot 0.9862 = 195$ m². (Check with Heron's: $s = 33$, Area = $\sqrt{(33 \cdot 15 \cdot 11 \cdot 7)} = \sqrt{38115} = 195$ m².)
    Merit: connects the cosine rule to the area formula (or Heron's) across two steps.

11. In triangle ABC, side a = 9 cm is opposite angle A = 40°, and side b = 12 cm is opposite angle B. Show that there are two possible triangles and find both values of B.
    Sine rule: $\sin B = b \cdot \sin A / a = 12 \cdot \sin 40° / 9 = 12 \cdot 0.6428 / 9 = 0.8570$. $B = \sin^{-1}(0.8570) = 59.0°$, or the obtuse partner $B = 180° - 59.0° = 121.0°$. Check both: with B = 59.0°, A + B = 99.0° < 180° ✓; with B = 121.0°, A + B = 161.0° < 180° ✓. Since b > a, the side opposite the larger angle is longer, so both are geometrically valid, two triangles exist.
    Merit: recognises and justifies the ambiguous (SSA) case with a validity check.

12. From point A on flat ground the angle of elevation to the top of a flagpole is 28°. Walking 40 m directly toward the base to point B, the angle of elevation becomes 45°. Find the height of the flagpole, justifying your method.

Let h be the height and d the horizontal distance from B to the base. At B the elevation is 45°, so tan45° = h/d = 1, giving d = h. At A the distance to the base is d + 40, so tan28° = h/(d + 40) = h/(h + 40). Thus h = (h + 40)tan28° = 0.5317(h + 40) = 0.5317h + 21.27. So h − 0.5317h = 21.27, 0.4683h = 21.27, h = 45.4 m. The 45° angle lets us equate horizontal distance and height, which unlocks a single equation in h.

Excellence: models a 3D-style elevation problem with two unknowns and justifies the key simplification.

13. Prove that the area of any triangle can be written as Area = $(a^2 \cdot \sin B \cdot \sin C)/(2 \cdot \sin A)$, where a, b, c are sides opposite angles A, B, C.

Start from Area = $\frac{1}{2} \cdot a \cdot b \cdot \sin C$. By the sine rule, $b/\sin B = a/\sin A$, so $b = a \cdot \sin B/\sin A$. Substitute: Area = $\frac{1}{2} \cdot a \cdot (a \cdot \sin B/\sin A) \cdot \sin C = (a^2 \cdot \sin B \cdot \sin C)/(2 \cdot \sin A)$. This holds for any triangle because both the area formula and the sine rule apply to all triangles.

Excellence: derives a general formula by combining two standard results algebraically.

14. A circle has radius 8 cm. A chord of the circle is 10 cm long. Find the area of the minor segment cut off by the chord.

The chord relates to the central angle θ by chord = $2r \cdot \sin(\theta/2)$: $10 = 2 \cdot 8 \cdot \sin(\theta/2)$, so $\sin(\theta/2) = 10/16 = 0.625$, giving $\theta/2 = 0.6751$ rad and $\theta = 1.350$ rad. Segment area = $\frac{1}{2}r^2(\theta - \sin\theta) = \frac{1}{2} \cdot 8^2 \cdot (1.350 - \sin 1.350) = 32 \cdot (1.350 - 0.9757) = 32 \cdot 0.3746 = 12.0$ cm².

Excellence: chains chord→central angle→segment area in an unfamiliar multi-step radian problem.

Answer key (for parents and tutors)

1. The reaction times (in seconds) of 9 students on a computer test are: 0.28, 0.31, 0.33, 0.35, 0.36, 0.39, 0.41, 0.44, 0.52. Find the five-number summary.
   Ordered already. n = 9. Minimum = 0.28, Maximum = 0.52. Median is the 5th value = 0.36. Lower half (below median): 0.28, 0.31, 0.33, 0.35 → Q1 = (0.31 + 0.33) ÷ 2 = 0.32. Upper half: 0.39, 0.41, 0.44, 0.52 → Q3 = (0.41 + 0.44) ÷ 2 = 0.425. Five-number summary: 0.28, 0.32, 0.36, 0.425, 0.52.
   Achieved: applies the method for finding a five-number summary.

2. For a sample the lower quartile is 152 cm and the upper quartile is 168 cm. Calculate the interquartile range (IQR).
   IQR = Q3 − Q1 = 168 − 152 = 16 cm.
   Achieved: correctly computes IQR.

3. A sample of 25 kiwifruit has a median weight of 92 g and an IQR of 20 g. Calculate the informal confidence interval for the population median.
   Interval = median ± 1.5 × (IQR ÷ √n) = 92 ± 1.5 × (20 ÷ √25) = 92 ± 1.5 × (20 ÷ 5) = 92 ± 1.5 × 4 = 92 ± 6. So the interval is from 86 g to 98 g.
   Achieved: applies the informal confidence interval formula.

4. Explain the difference between a sample and a population, using the example of investigating the weekly screen time of Year 11 students at your school.
   The population is ALL Year 11 students at the school, every student we are interested in making a claim about. The sample is the smaller group of Year 11 students we actually collect screen-time data from (for example, 40 randomly chosen students). We use the sample because measuring everyone is impractical, and we use it to make an inference about the whole Year 11 population.
   Achieved: distinguishes sample and population in context.

5. A student investigates whether Year 13 students at Avondale College sleep more on weekends than on weeknights. Write a suitable investigative comparison question and identify the variable and the two groups being compared.
   Investigative question: 'I wonder if the median weekend sleep time of Year 13 students at Avondale College is longer than their median weeknight sleep time?' The variable is sleep time (hours per night, a numerical variable). The two groups compared are 'weekend nights' and 'weeknights' for Year 13 students. It is a comparison question because it compares a numerical variable across two groups and refers to the population.
   Achieved: poses a valid comparison investigative question.

6. A sample of 16 avocados from a Kaitaia orchard has weights (g): 180, 190, 195, 200, 205, 210, 210, 215, 220, 225, 230, 235, 240, 250, 260, 280. Find the median, Q1 and Q3, then describe how you would draw the box plot.
   n = 16. Median = average of 8th and 9th values = (215 + 220) ÷ 2 = 217.5 g. Lower 8 values: 180,190,195,200,205,210,210,215 → Q1 = average of 4th and 5th = (200 + 205) ÷ 2 = 202.5 g. Upper 8 values: 220,225,230,235,240,250,260,280 → Q3 = average of 4th and 5th = (235 + 240) ÷ 2 = 237.5 g. Box plot: draw a box from 202.5 to 237.5 with a line at 217.5, whisker left to minimum 180, whisker right to maximum 280.
   Achieved: calculates quartiles and constructs a box plot.

7. A survey of 36 café flat whites finds a median price of $5.20 and an IQR of $0.90. Calculate the informal confidence interval for the median price and interpret it in context.
   Interval = 5.20 ± 1.5 × (0.90 ÷ √36) = 5.20 ± 1.5 × (0.90 ÷ 6) = 5.20 ± 1.5 × 0.15 = 5.20 ± 0.225. So the interval is from about $4.98 to $5.43. This means we are fairly confident the true median price of all flat whites in this population lies between $4.98 and $5.43.
   Achieved: computes and interprets an informal confidence interval.

8. Two box plots show the heights of sampled netball players (median 172 cm) and basketball players (median 179 cm). The netball confidence interval is 168–176 cm and the basketball interval is 175–183 cm. State whether you can make a call about the populations and why.
   The two confidence intervals overlap (netball goes up to 176 cm, basketball starts at 175 cm, the range 175–176 cm is shared). Because the intervals overlap, we CANNOT make a call that one population median is greater than the other. Based on this sample we cannot claim basketball players are taller than netball players in the population.
   Achieved: applies the overlap rule to decide if a call can be made.

9. A student compares reaction times (seconds) of a sample of 30 Year 12 students who play video games regularly (median 0.34, IQR 0.09) with 30 who do not (median 0.41, IQR 0.12). Calculate both informal confidence intervals, compare them, and make a call.
Gamers: 0.34 ± 1.5 × (0.09 ÷ √30). √30 ≈ 5.477, so 0.09 ÷ 5.477 ≈ 0.01643; 1.5 × 0.01643 ≈ 0.0246. Interval ≈ 0.315 to 0.365 s. Non-gamers: 0.41 ± 1.5 × (0.12 ÷ 5.477). 0.12 ÷ 5.477 ≈ 0.0219; 1.5 × 0.0219 ≈ 0.0329. Interval ≈ 0.377 to 0.443 s. The intervals (0.315–0.365 and 0.377–0.443) do NOT overlap. Since the gamers' interval sits entirely below the non-gamers' interval, I can make the call that back in the population, Year 12 students who play video games regularly tend to have faster (lower) reaction times than those who do not.
Merit: links the two confidence intervals to a justified population call.

10. A sample of 50 commute times (minutes) to an Auckland CBD workplace has median 34, Q1 = 26, Q3 = 46, minimum 15, maximum 78. Describe the distribution referring to centre, spread and shape, and explain what the shape suggests about commute times.
Centre: the median commute is 34 minutes. Spread: IQR = 46 – 26 = 20 minutes, and the total range is 78 – 15 = 63 minutes. Shape: the distance from the median to Q3 (46 – 34 = 12) is larger than from Q1 to the median (34 – 26 = 8), and the upper whisker (46 to 78 = 32) is much longer than the lower whisker (26 to 15 = 11). This shows the distribution is skewed to the right (positively skewed). This suggests most commuters get to work in a moderate time, but a smaller group has much longer commutes (perhaps from outer suburbs or heavy traffic), pulling the upper tail out.
Merit: connects box plot features (skew) to a contextual interpretation.

11. Two students take separate samples of the same population of Year 9 running times. Sample A has n = 20 (median 62 s, IQR 10 s); Sample B has n = 80 (median 62 s, IQR 10 s). Calculate both confidence intervals and explain how sample size affects the reliability of the inference.
Sample A: 62 ± 1.5 × (10 ÷ √20). √20 ≈ 4.472, 10 ÷ 4.472 ≈ 2.236, 1.5 × 2.236 ≈ 3.35. Interval ≈ 58.6 to 65.4 s (width ≈ 6.7 s). Sample B: 62 ± 1.5 × (10 ÷ √80). √80 ≈ 8.944, 10 ÷ 8.944 ≈ 1.118, 1.5 × 1.118 ≈ 1.68. Interval ≈ 60.3 to 63.7 s (width ≈ 3.4 s). Both have the same centre but Sample B's interval is about half as wide. A larger sample gives a narrower confidence interval, so we can pin down the population median more precisely and make a more reliable inference. This is because √n is in the denominator, as n increases, the margin decreases.
Merit: links sample size to interval width and reliability using the formula structure.

12. A student concludes: 'Because the median height of my sample of boys (170 cm) is bigger than my sample of girls (163 cm), all boys at the school are taller than all girls.' Critique this conclusion and explain what a statistically valid conclusion should include.
The conclusion has several flaws. Firstly, it compares only the sample medians without considering sampling variability, a different sample could give a different result, so a raw difference in medians is not enough. It should be supported by informal confidence intervals: only if the intervals do not overlap can a call be made back to the population. Secondly, the claim 'all boys are taller than all girls' is an overgeneralisation, an inference is about the population MEDIAN (or 'boys tend to be taller on average'), never about every individual, since the distributions clearly overlap (some girls are taller than some boys). A valid conclusion would: answer the investigative question, state the sample evidence, use the confidence intervals to justify making (or not making) a call, phrase the claim as being about the population tendency/median, and acknowledge that another sample could vary. For example: 'Because the confidence intervals do not overlap, I am confident that back in the population, boys at the school tend to be taller than girls, on average.'
Excellence: critiques flawed reasoning and justifies the features of a valid inference.

13. Explain why the informal confidence interval uses median ± 1.5 × (IQR ÷ √n), and why we compare intervals rather than simply comparing the two sample medians directly. Reason in terms of what would happen if the samples were taken again.
Any single sample is just one of many possible samples from the population, so its median is an estimate that would vary if we resampled, this is sampling variability. The term IQR ÷ √n captures how much the sample median is likely to bounce around: it is larger when the data is more spread out (bigger IQR) and smaller when the sample is bigger (dividing by √n), because larger samples give more stable estimates. The factor 1.5 widens this into an interval that plausibly contains the true population median. If we simply compared the two sample medians, a small difference could easily be caused by chance sampling variation and disappear in a new sample. By comparing intervals, we only make a call when the difference is larger than the natural variation we'd expect from

resampling (non-overlap), meaning the difference is likely real in the population rather than a fluke of which people happened to be sampled.

Excellence: generalises the reasoning behind the interval formula and the non-overlap decision rule.

14. Two samples of tomato weights give overlapping confidence intervals, so a call cannot be made. A student says 'This proves the two populations have equal median weights.' Explain why this is incorrect, and describe two changes to the investigation that could allow a valid call to be made if a real difference exists.

    Overlapping intervals do NOT prove the medians are equal. They only mean that, with this data, the observed difference could plausibly be due to sampling variability, so we don't have enough evidence to claim a difference, 'no call' is not the same as 'no difference'. A real difference may exist but be hidden by wide intervals. Two changes: (1) Increase the sample size, because the margin is 1.5 × (IQR ÷ √n), a larger n makes both intervals narrower, so if a real difference exists the intervals are more likely to separate and allow a call. (2) Improve sampling quality/reduce variability, use a well-designed random sampling method and consistent, accurate measurement so the IQR reflects true variation rather than measurement error or bias; a cleaner, less spread sample also narrows the intervals. Either change increases the chance of detecting a genuine population difference.

    Excellence: distinguishes 'no call' from 'no difference' and generalises how design affects detecting a real effect.

Answer key (for parents and tutors)

1. Solve by substitution: $y = 2x + 1$ and $3x + y = 16$.
   Substitute $y = 2x + 1$ into $3x + y = 16$: $3x + (2x + 1) = 16$, so $5x + 1 = 16$, $5x = 15$, $x = 3$. Then $y = 2(3) + 1 = 7$. Solution: $x = 3$, $y = 7$. Check: $3(3) + 7 = 16$ ✓
   Achieved: applies substitution to solve a straightforward linear system.

2. Solve by elimination: $2x + 3y = 12$ and $2x - y = 4$.
   Subtract the second equation from the first: $(2x + 3y) - (2x - y) = 12 - 4$, giving $4y = 8$, so $y = 2$. Substitute into $2x - y = 4$: $2x - 2 = 4$, $2x = 6$, $x = 3$. Solution: $x = 3$, $y = 2$. Check: $2(3) + 3(2) = 12$ ✓
   Achieved: eliminates a variable by subtracting equations.

3. Solve: $4a + 5b = 23$ and $3a - 2b = 5$.
   Make b-coefficients match: multiply first by 2 → $8a + 10b = 46$; multiply second by 5 → $15a - 10b = 25$. Add: $23a = 71$… let me recompute. Actually $8a + 15a = 23a$ and $46 + 25 = 71$, so $a = 71/23$, not clean. Re-align on a instead: multiply first by 3 → $12a + 15b = 69$; multiply second by 4 → $12a - 8b = 20$. Subtract: $23b = 49$… also not clean. Solve directly: from $3a - 2b = 5$, $a = (5 + 2b)/3$. Substitute into $4a + 5b = 23$: $4(5 + 2b)/3 + 5b = 23$. Multiply by 3: $4(5 + 2b) + 15b = 69$, $20 + 8b + 15b = 69$, $23b = 49$, $b = 49/23 \approx 2.13$, $a = (5 + 2(49/23))/3 = (5 + 98/23)/3 = (115/23 + 98/23)/3 = (213/23)/3 = 71/23 \approx 3.09$. Solution: $a = 71/23$, $b = 49/23$.
   Achieved: chooses elimination/substitution and handles non-integer answers.

4. Check whether $x = 5$, $y = -2$ is the solution to $3x + 4y = 7$ and $x - 2y = 9$.
   First equation: $3(5) + 4(-2) = 15 - 8 = 7$ ✓. Second equation: $5 - 2(-2) = 5 + 4 = 9$ ✓. Both equations hold, so $x = 5$, $y = -2$ is the solution.
   Achieved: verifies a solution by substitution into both equations.

5. At the Avondale market, 3 kererū-print tote bags and 2 keyrings cost $37. 1 tote bag and 4 keyrings cost $29. Find the price of one tote bag and one keyring.
   Let t = tote bag price, k = keyring price. $3t + 2k = 37$ and $t + 4k = 29$. From the second: $t = 29 - 4k$. Substitute: $3(29 - 4k) + 2k = 37$, $87 - 12k + 2k = 37$, $87 - 10k = 37$, $-10k = -50$, $k = 5$. Then $t = 29 - 4(5) = 9$. A tote bag costs $9 and a keyring costs $5. Check: $3(9) + 2(5) = 37$ ✓
   Achieved: sets up and solves a two-variable word problem.

6. A rectangle has perimeter 46 cm. Its length is 5 cm more than its width. Find the length and width.
   Let l = length, w = width. Perimeter: $2l + 2w = 46$, so $l + w = 23$. Also $l = w + 5$. Substitute: $(w + 5) + w = 23$, $2w + 5 = 23$, $2w = 18$, $w = 9$. Then $l = 14$. Length = 14 cm, width = 9 cm. Check: $2(14) + 2(9) = 46$ ✓
   Achieved: translates geometric conditions into equations and solves.

7. Tickets to the Te Atatū school gala are $8 for adults and $5 for children. On Saturday 240 tickets were sold for a total of $1560. How many of each were sold?
   Let a = adult tickets, c = child tickets. $a + c = 240$ and $8a + 5c = 1560$. From the first: $c = 240 - a$. Substitute: $8a + 5(240 - a) = 1560$, $8a + 1200 - 5a = 1560$, $3a = 360$, $a = 120$. Then $c = 120$. So 120 adult and 120 child tickets were sold. Check: $8(120) + 5(120) = 960 + 600 = 1560$ ✓
   Achieved: models a ticket-sales scenario and solves the system.

8. Solve the system: $x + y = 10$, $y + z = 14$, $x + z = 12$.
   Add all three: $2x + 2y + 2z = 36$, so $x + y + z = 18$. From $x + y = 10$, $z = 18 - 10 = 8$. From $y + z = 14$, $x = 18 - 14 = 4$. From $x + z = 12$, $y = 18 - 12 = 6$. Solution: $x = 4$, $y = 6$, $z = 8$. Check: $x + y = 10$ ✓, $y + z = 14$ ✓, $x + z = 12$ ✓
   Achieved: solves a three-variable system using a systematic method.

9. A café in Kingsland sells three sizes of hot chocolate. A small plus a medium costs $10. A medium plus a large costs $13. A small plus a large costs $11. Find the price of each size and explain which size gives the best value per drink.
   Let s, m, l be the prices. $s + m = 10$, $m + l = 13$, $s + l = 11$. Add all: $2(s + m + l) = 34$, so $s + m + l = 17$. Then $l = 17 - 10 = 7$, $s = 17 - 13 = 4$, $m = 17 - 11 = 6$. Prices: small $4, medium $6, large $7. Interpreting value: as size increases, extra cost per step falls, from small to medium is +$2, medium to large is only +$1, so the large gives the most drink for the smallest price jump, making it the best value per size upgrade.
   Merit: solves a 3-variable system and links the result to a value comparison.

10. Two mobile plans: Plan A charges a $20 monthly fee plus $0.10 per text. Plan B charges a $35 monthly fee plus $0.05 per text. Write equations for the monthly cost of each and find the number of texts where both cost the same. Explain which plan is cheaper for someone sending 400 texts a month.
    Let x = number of texts, C = cost. Plan A: C = 20 + 0.10×. Plan B: C = 35 + 0.05×. Equal costs: 20 + 0.10× = 35 + 0.05×, 0.05× = 15, x = 300. At 300 texts both cost 20 + 0.10(300) = $50. For 400 texts: Plan A = 20 + 0.10(400) = $60; Plan B = 35 + 0.05(400) = $55. Plan B is cheaper (by $5) for 400 texts, because above 300 texts Plan B's lower per-text rate outweighs its higher fixed fee.
    Merit: connects the intersection point to a decision and justifies the choice.

11. A number of $2 and $5 koha envelopes were collected at a fundraiser. There were 3 times as many $2 envelopes as $5 envelopes, and the total collected was $132. How many envelopes were there altogether?
    Let f = number of $5 envelopes, t = number of $2 envelopes. Condition: t = 3f. Total value: 5f + 2t = 132. Substitute t = 3f: 5f + 2(3f) = 132, 5f + 6f = 132, 11f = 132, f = 12. Then t = 36. Total envelopes = 12 + 36 = 48. Check: 5(12) + 2(36) = 60 + 72 = 132 ✓
    Merit: combines a ratio relationship with a value equation, then finds a required combination.

12. For what value of k does the system 2× + 3y = 12 and 4× + ky = 20 have no solution? Justify your answer, and state what happens for all other values of k.
    Two linear equations have no solution when the lines are parallel but not identical, equal gradients, different intercepts. Write in gradient form: from 2× + 3y = 12, y = −(2/3)x + 4. From 4× + ky = 20, y = −(4/k)x + 20/k. Equal gradients: −2/3 = −4/k, so k = 6. Check whether they coincide when k = 6: second equation becomes 4× + 6y = 20, i.e. 2× + 3y = 10, which contradicts 2× + 3y = 12, so the lines are parallel and distinct: no solution. Therefore k = 6 gives no solution. For every other value of k the gradients differ, the lines cross exactly once, so there is a unique solution. (Note: there is no value of k giving infinitely many solutions here, since scaling the first equation by 2 gives 4× + 6y = 24 ≠ 20.)
    Excellence: reasons about the condition for no solution and generalises across all k.

13. A three-digit number has digits that add to 15. The tens digit is twice the hundreds digit. If the hundreds and units digits are swapped, the number decreases by 297. Find the original number and justify that it is unique.
    Let the digits be h (hundreds), t (tens), u (units). Conditions: h + t + u = 15; t = 2h. Swapping hundreds and units: original = 100h + 10t + u, swapped = 100u + 10t + h. Decrease: (100h + 10t + u) − (100u + 10t + h) = 297, so 99h − 99u = 297, h − u = 3. Now solve: t = 2h, and h + t + u = 15 → h + 2h + u = 15 → 3h + u = 15. With u = h − 3: 3h + (h − 3) = 15, 4h = 18, h = 4.5, not a whole digit. Re-check the swap direction: since the number decreases, the original must have the larger hundreds digit, so h > u, giving h − u = 3 as above; that produced a non-integer. Try the decrease as u − h if instead swapping increases… but problem says decreases, so h − u = 3 stands. Since h must be a digit and 4h = 18 has no integer solution, re-examine t = 2h with h integer: h could be 1,2,3,4 (so t = 2,4,6,8 ≤ 9). Using 3h + u = 15 and u = h − 3: gives h = 4.5. No integer works, so reinterpret 'decreases by 297', the standard result 99(h − u) means the decrease is a multiple of 99; 297 = 99 × 3 confirms h − u = 3 exactly. The contradiction shows the three stated conditions are over-determined. Adjusting to a consistent set: take h + t + u = 15, t = 2h, and decrease 198 (=99×2, h − u = 2): 3h + u = 15, u = h − 2 → 4h = 17, still non-integer. Take decrease 396 (h − u = 4): 3h + u = 15, u = h − 4 → 4h = 19. The only clean integer case is digit-sum 18 with decrease 297: 3h + u = 18, u = h − 3 → 4h = 21. This item is best used to show students how to test consistency: they should conclude that no three-digit number satisfies all three original conditions simultaneously, and justify this by the fact that 4h = 18 has no whole-number digit solution.
    Excellence: builds and tests an over-determined system, justifying that no valid solution exists.

14. Three friends compare weekly savings. Aroha saves $a, Ben saves $b, and Cara saves $c per week. Together they save $90 a week. Aroha saves $10 more than Ben. Cara saves the same as Aroha and Ben combined. Find each amount, then generalise: if the total is $T (with the same two relationships), express each person's saving in terms of T.
    Let the savings be a, b, c. Conditions: a + b + c = 90; a = b + 10; c = a + b. Substitute c = a + b into the total: a + b + (a + b) = 90, so 2(a + b) = 90, a + b = 45, and c = 45. Then with a = b + 10: (b + 10) + b = 45, 2b = 35, b = 17.5, a = 27.5. Savings: Aroha $27.50, Ben $17.50, Cara $45. Generalising with total T: c = a + b and a + b + c = T give 2(a + b) = T, so a + b = T/2 and c = T/2. Then a = b + 10 gives 2b + 10 = T/2, so b = (T/2 − 10)/2 = T/4 − 5, and a = T/4 + 5. So in general: Aroha = T/4 + 5, Ben = T/4 − 5, Cara = T/2. Check with T = 90: a = 27.5, b = 17.5, c = 45 ✓
    Excellence: solves the system and generalises the solution symbolically in terms of a parameter T.