Vertex Academy

NCEA Level 1 Maths workbook

4 worksheets · answers at the back

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NCEA Level 1 · AS 91947: Mathematical Reasoning (Number & Algebra)

Name: _____ Date: _____

Kia ora! This standard is all about showing clear algebraic reasoning, solving equations, working with expressions, and generalising patterns. Work through each section in order, always writing out your steps, because in the exam your reasoning earns the marks, not just the final answer.

Revision summary

- Expanding: $a(b + c) = ab + ac$; $(x + p)(x + q) = x^2 + (p + q)x + pq$.
- Factorising: common factor first, then quadratics $x^2 + bx + c = (x + p)(x + q)$ where $p + q = b$ and $pq = c$. Difference of two squares: $a^2 - b^2 = (a + b)(a - b)$.
- Solving linear equations: do the same operation to both sides to isolate the variable. For quadratics, factorise and use the null factor law (if $AB = 0$ then $A = 0$ or $B = 0$).
- Simultaneous equations: use substitution or elimination to find the point that satisfies both equations.
- Exponent laws: $x^a \times x^b = x^{a+b}$; $x^a \div x^b = x^{a-b}$; $(x^a)^b = x^{ab}$; $x^0 = 1$; $x^{-a} = 1/x^a$.
- Inequalities: solve like equations, but reverse the sign (< becomes >) when multiplying or dividing by a negative.
- Proportional reasoning: set up equivalent ratios or a rate, then scale up or down.
- Generalising a linear pattern: term = (common difference) × n + (constant); always justify why the rule works from the structure of the pattern.

Key skills

1. Expand and simplify: $3(2x - 5) + 4x$.

2. Factorise fully: $x^2 + 7x + 12$.

3. Solve for x: $5x - 8 = 27$.

4. Simplify: $(2a^3)^2 \times a^4$.

Achieved

5. Aroha buys 3 iceblocks and a $2 juice at the Avondale dairy and pays $9.50. Write and solve an equation to find the price of one iceblock.

6. Solve the quadratic equation $x^2 - 5x - 14 = 0$.

7. Solve the inequality $4x + 3 \leq 19$ and describe the solution.

8. A recipe for 4 people needs 600 g of kūmara. How much kūmara is needed for 10 people?

Merit

9. A rectangle has length 3 cm more than its width. Its area is 40 cm². Set up and solve an equation to find the dimensions.

10. Solve the simultaneous equations $2x + y = 11$ and $3x - y = 4$, and explain what the solution represents.

11. Simplify the expression $(x^2 - 9) \div (x + 3)$, showing your reasoning.

Excellence

12. A pattern of square tiles grows so that stage 1 uses 4 tiles, stage 2 uses 7 tiles, stage 3 uses 10 tiles. Find a rule for the number of tiles at stage n, justify why it works, and determine which stage uses 100 tiles.

13. Prove that the sum of any three consecutive whole numbers is always a multiple of 3.

14. For what values of k does the equation $x^2 + kx + 9 = 0$ have exactly one solution? Justify your answer.

NCEA Level 1 · AS 91944: Exploring Data with the Statistical Enquiry Cycle

Name: Date:

Hey! This standard is all about running a proper statistical investigation from start to finish, asking a good question, sorting your data, making sensible graphs, and writing conclusions that the numbers actually support. Work through each section in order, and always back up what you say with evidence from your data.

Revision summary

- The statistical enquiry cycle (PPDAC): Problem (pose a comparison/summary question), Plan (how to get data), Data (collect/clean/source), Analysis (graphs + summary stats), Conclusion (answer the question with evidence).
- A good comparison question names the two groups, the numeric variable, and asks which tends to be bigger, e.g. 'Do Year 11 boys tend to be taller than Year 11 girls at our school?'
- Measures of centre: mean = sum of values ÷ number of values; median = middle value when ordered; mode = most common value.
- Measures of spread: range = max − min; interquartile range IQR = upper quartile (Q3) − lower quartile (Q1). IQR describes the middle 50% and resists outliers.
- Shape: describe as symmetric, right-skewed (tail to high values) or left-skewed (tail to low values); note any clusters, gaps or outliers.
- Best displays: dot plot or box plot for one numeric variable (great for comparing two groups), bar chart for categories, back-to-back or paired displays for comparisons.
- Comparing groups: compare medians (centre), overlap of boxes/IQRs (spread), and shape. If the difference in medians is large relative to the spread, you can make a stronger 'back to the population' claim.
- Conclusions must be evidence-based: quote actual values from your graphs/stats, and only generalise beyond the sample cautiously if the data was collected sensibly.

Key skills

1. The number of texts sent by 9 students at Avondale College in one hour: 3, 5, 5, 8, 10, 12, 14, 18, 22. Find the median and the range.

2. Calculate the mean of these weekend earnings (in $): 40, 55, 60, 60, 85.

3. For the data 12, 15, 15, 18, 20, 21, 25, 30, find the lower quartile (Q1), upper quartile (Q3) and the interquartile range.

4. A dot plot of daily bus wait times (minutes) has most dots between 2 and 6, with one dot far out at 20. Describe the shape and identify any outlier.

Achieved

5. A student wants to investigate reaction times at Vertex Academy. Write a suitable investigative comparison question, and state what data would need to be collected.

6. Given the shoe sizes of 10 netball players: 6, 7, 7, 8, 8, 8, 9, 9, 10, 11, choose a suitable display and describe the shape and centre.

7. The times (minutes) for 12 people to complete a 3 km parkrun in New Lynn: 16, 18, 18, 19, 20, 21, 22, 23, 24, 25, 27, 30. Find the median and the IQR.

8. A café records the number of coffees sold each hour: 14, 22, 18, 30, 25, 19. Calculate the mean and state whether the mean or median better represents a 'typical' hour if one hour had a huge one-off order of 90 coffees added.

Merit

9. Two Year 11 classes did the same 40-mark test. Class A: median 26, Q1 20, Q3 31. Class B: median 24, Q1 22, Q3 27. Compare the two distributions, linking centre and spread, and comment on which class did better.

10. A student collected commuting distances (km) from a survey posted only in the school's cycling club group chat, then concluded 'most students at our school live within 5 km of school'. Explain two problems with this conclusion and how the plan could be improved.

11. Weekly pocket money ($) for a sample: 5, 10, 10, 12, 15, 15, 15, 20, 25, 60. A student says 'the average is $18.70 so a typical student gets about $19'. Using the shape of the data, evaluate this claim and suggest a better summary.

Excellence

12. Two box plots comparing sleep hours of Year 9 and Year 13 students overlap: Year 9 median 8.5 h (IQR 7.5–9.5), Year 13 median 6.5 h (IQR 5.5–7.5). Explain, with justification, whether it is reasonable to claim that in the wider student population Year 13 students tend to sleep less than Year 9 students.

13. Prove that adding a single new value equal to the current mean of a data set leaves the mean unchanged, and explain what this shows about the mean's sensitivity.

14. A student surveyed 200 people at Avondale train station one Monday morning and found the median daily screen time was 1.5 hours, concluding 'New Zealanders spend about 1.5 hours per day on screens'. Critically evaluate every stage of this enquiry (problem, plan, data, analysis, conclusion) and describe a redesign that would support a valid national claim.

NCEA Level 1 · AS 91945: Maths in Aotearoa: Rates, Ratios & Money

Name: Date:

Hi! This standard is all about using number and algebra methods, rates, ratios, percentages and financial maths, to explore real-life problems in Aotearoa and the Pacific. Work through each section in order, always show your method clearly, and check your answer makes sense in the context.

Revision summary

- Percentage of an amount: amount × (percent ÷ 100). To increase by x%: multiply by (1 + x/100); to decrease: multiply by (1 − x/100).
- Reverse percentage: original = final ÷ (multiplier). E.g. price after 15% GST added: original = total ÷ 1.15.
- Rate = quantity ÷ time (or per unit). Keep units consistent, e.g. km/h, $/kg, L/100km.
- Ratio sharing: add ratio parts, divide total by parts to find one part, then multiply. a:b means split into (a+b) equal shares.
- Unit rate for comparison ('best buy'): divide price ÷ quantity to get cost per unit, then compare.
- Simple interest: I = P × r × t (r as a decimal, t in years). Amount = P + I.
- Currency conversion: multiply by the exchange rate; divide to convert back.
- Always state units, round sensibly (money to the nearest cent), and write a clear concluding sentence answering the question.

Key skills

1. Find 15% of $240.

2. Share $600 between two marae committees in the ratio 3 : 2.

3. A car travels 180 km in 2.5 hours. Find its average speed in km/h.

4. A 2 kg bag of kūmara costs $9.60. What is the cost per kilogram?

Achieved

5. A pair of gumboots is priced at $65 before GST. GST of 15% is added at the checkout. What is the total price?

6. At a Pasifika festival, a food stall sells 3 items for $12. At this rate, how much would 8 items cost?

7. Aroha deposits $800 in an account paying 4% simple interest per year. How much interest does she earn in 3 years?

8. A recipe for a hāngī feeds 6 people and uses 1.5 kg of pork. How much pork is needed to feed 20 people?

Merit

9. A shop reduces the price of a surfboard by 20% in a sale, making the sale price $360. What was the original price, and how many dollars were saved?

10. Two brands of coconut milk are compared. Brand A: 400 mL for $3.20. Brand B: 600 mL for $4.50. Which is better value, and by how much per litre?

11. Tane is driving from Auckland to Taupō, a distance of 280 km. His car uses fuel at 7.5 L per 100 km, and petrol costs $2.80 per litre. Calculate the total fuel cost for the trip.

Excellence

12. A tourist changes $500 NZD to Fijian dollars at a rate of 1 NZD = 1.35 FJD, paying a $10 NZD flat fee first. In Fiji she spends 600 FJD, then changes the rest back at 1 NZD = 1.40 FJD. How many NZD does she end up with? Explain why she loses money overall even though nothing was 'spent' on the conversions themselves.

13. Two towns run water tanks. Town A's tank fills at a steady 15 L per minute; Town B's fills at 12 L per minute but already holds 900 L when both start. After how many minutes do the tanks hold the same volume, and prove your answer using an equation.

14. A koha (donation) fund grows by the same percentage each year. After 2 years $2000 has grown to $2205. Find the annual percentage growth rate, and use it to predict the value after 3 years, explaining any assumption.

NCEA Level 1 · AS 91946: Making Sense of Maths in Context

Name: _____ Date: _____

This standard is all about reading real information, graphs, tables, formulae and models, then using it to answer questions and back up your thinking. Work through each section in order, and always ask yourself 'what does this number actually mean here?'

Revision summary

- Read carefully: check axis labels, units, scales, headings and the key before you calculate.
- To apply a formula, substitute the given values, keep units consistent, then solve step by step.
- Rate = amount ÷ time (e.g. $/hour, km/h). Cost models often look like: total = fixed charge + rate × quantity.
- Percentage change = (new − old) ÷ old × 100. To increase by r%, multiply by (1 + r/100).
- Reading a graph: interpolate between points; a steeper line means a faster rate of change.
- Justify conclusions with evidence, quote actual numbers from the source, and state any assumptions.
- For Merit, link two or more pieces of information together (e.g. compare two plans, connect a table to a graph).
- For Excellence, generalise (write a rule), prove why a claim is true or false, or evaluate which option is best and defend it.

Key skills

1. A water tank fills at a steady rate. After 3 minutes it holds 240 litres. What is the filling rate in litres per minute?

2. A jacket priced at $80 is discounted by 25%. What is the sale price?

3. A phone plan costs $15 per month plus $0.10 per text. Write a formula for the total cost C (in dollars) for t texts in a month.

4. A line graph of a car's distance shows it travelled 150 km in 2 hours at a steady speed. What was its speed?

Achieved

5. Aroha earns $22.50 per hour at a café in Avondale. One week she works 18 hours. Using the model Pay = rate × hours, calculate her pay for the week.

6. A table shows daily rainfall in Auckland: Mon 4 mm, Tue 0 mm, Wed 12 mm, Thu 8 mm, Fri 6 mm. What was the total rainfall, and the mean daily rainfall over the five days?

7. A recipe for 4 people needs 600 g of flour. Kiri is cooking for 10 people. How much flour does she need?

8. A courier charges a $6 base fee plus $2.50 per kilometre. Use C = 6 + 2.50d to find the cost of a 14 km delivery.

Merit

9. Two gyms in West Auckland offer memberships. FitZone charges $40 joining fee plus $18 per month. PowerHouse charges no joining fee but $22 per month. After how many months do the two gyms cost the same amount, and what is that amount?

10. A shop increases a $250 bike price by 20% for the summer, then in autumn reduces the new price by 20%. A customer says 'the price is back to $250'. Show whether the customer is correct.

11. A graph shows a plant's height: 5 cm at week 0, growing steadily to 25 cm at week 4. Write a formula for the height h after w weeks, then use it to predict the height at week 7. State one reason the prediction may be unreliable.

Excellence

12. A phone company offers Plan A: $30 flat per month for unlimited data. Plan B: $10 per month plus $2 per GB. For what range of data usage is Plan A the cheaper choice? Justify your answer with a general argument.

13. A claim in a local newsletter states: 'Averaging 90 km/h over a 180 km trip means you can drive 100 km/h for the first half of the distance and 80 km/h for the second half.' Determine whether this claim is true, showing your reasoning.

14. A container's rental cost is modelled by C = 50 + 8d for d days. A rival offers a weekly rate of $90 per 7-day week (part weeks rounded up to a full week). Explain, with justification, for which numbers of days each option is cheaper.

Answer key (for parents and tutors)

1. Expand and simplify: $3(2x - 5) + 4x$.
   $3(2x - 5) = 6x - 15$. Adding $4x$ gives $6x - 15 + 4x = 10x - 15$.
   Achieved fluency: applies the distributive law and collects like terms.

2. Factorise fully: $x^2 + 7x + 12$.
   Find two numbers with product 12 and sum 7: these are 3 and 4. So $x^2 + 7x + 12 = (x + 3)(x + 4)$.
   Achieved fluency: factorises a monic quadratic.

3. Solve for x: $5x - 8 = 27$.
   Add 8 to both sides: $5x = 35$. Divide by 5: $x = 7$.
   Achieved fluency: solves a two-step linear equation.

4. Simplify: $(2a^3)^2 \times a^4$.
   $(2a^3)^2 = 4a^6$. Then $4a^6 \times a^4 = 4a^{10}$ (add the exponents).
   Achieved fluency: applies exponent laws with a coefficient.

5. Aroha buys 3 iceblocks and a $2 juice at the Avondale dairy and pays $9.50. Write and solve an equation to find the price of one iceblock.
   Let the iceblock price be x. Then $3x + 2 = 9.50$. Subtract 2: $3x = 7.50$. Divide by 3: $x = 2.50$. Each iceblock costs $2.50.
   Achieved: forms and solves a linear equation from a word context.

6. Solve the quadratic equation $x^2 - 5x - 14 = 0$.
   Two numbers with product -14 and sum -5 are -7 and 2. So $(x - 7)(x + 2) = 0$. By the null factor law, $x = 7$ or $x = -2$.
   Achieved: factorises and solves a quadratic equation.

7. Solve the inequality $4x + 3 \leq 19$ and describe the solution.
   Subtract 3: $4x \leq 16$. Divide by 4: $x \leq 4$. The solution is all values of x less than or equal to 4.
   Achieved: solves a linear inequality.

8. A recipe for 4 people needs 600 g of kūmara. How much kūmara is needed for 10 people?
   Kūmara per person = $600 \div 4 = 150$ g. For 10 people: $150 \times 10 = 1500$ g (1.5 kg).
   Achieved: uses proportional reasoning via a unit rate.

9. A rectangle has length 3 cm more than its width. Its area is 40 cm². Set up and solve an equation to find the dimensions.
   Let the width be w. Then length = $w + 3$ and area = $w(w + 3) = 40$. Expand: $w^2 + 3w = 40$, so $w^2 + 3w - 40 = 0$. Factorise: two numbers with product -40 and sum 3 are 8 and -5, giving $(w + 8)(w - 5) = 0$. So $w = -8$ or $w = 5$. Width cannot be negative, so $w = 5$ cm and length = 8 cm.
   Merit: links a geometric context to a quadratic and rejects the invalid solution.

10. Solve the simultaneous equations $2x + y = 11$ and $3x - y = 4$, and explain what the solution represents.
    Add the two equations to eliminate y: $(2x + y) + (3x - y) = 11 + 4$, so $5x = 15$, $x = 3$. Substitute into the first equation: $2(3) + y = 11$, so $y = 5$. The solution (3, 5) is the point where the two straight lines cross.
    Merit: solves simultaneously and connects the answer to a graphical meaning.

11. Simplify the expression $(x^2 - 9) \div (x + 3)$, showing your reasoning.
    $x^2 - 9$ is a difference of two squares: $x^2 - 9 = (x + 3)(x - 3)$. So $(x + 3)(x - 3) \div (x + 3) = x - 3$, provided $x \neq -3$ (so we are not dividing by zero).
    Merit: connects difference-of-squares factorising with cancelling and notes the restriction.

12. A pattern of square tiles grows so that stage 1 uses 4 tiles, stage 2 uses 7 tiles, stage 3 uses 10 tiles. Find a rule for the number of tiles at stage n, justify why it works, and determine which stage uses 100 tiles.
    The numbers 4, 7, 10 increase by 3 each stage, so the rule is linear: tiles = $3n + c$. At $n = 1$, $3(1) + c = 4$, so $c = 1$. Rule: tiles = $3n + 1$. This works because each new stage adds a fixed group of 3 tiles to a starting core of 1 tile. Setting $3n + 1 = 100$ gives $3n = 99$, $n = 33$. Stage 33 uses 100 tiles.
    Excellence: generalises a pattern, justifies the rule from structure, and solves in reverse.

13. Prove that the sum of any three consecutive whole numbers is always a multiple of 3.
    Let the three consecutive numbers be n, n + 1 and n + 2. Their sum is n + (n + 1) + (n + 2) = 3n + 3 = 3(n + 1). Since 3(n + 1) has a factor of 3, the sum is always a multiple of 3 for any whole number n.
    Excellence: constructs a general algebraic proof of a number property.

14. For what values of k does the equation $x^2 + kx + 9 = 0$ have exactly one solution? Justify your answer.
    A quadratic has exactly one solution when it is a perfect square, i.e. when the discriminant $b^2 - 4ac = 0$. Here a = 1, b = k, c = 9, so $k^2 - 4(1)(9) = 0$, giving $k^2 = 36$, so k = 6 or k = −6. Check: for k = 6, $x^2 + 6x + 9 = (x + 3)^2 = 0$ gives the single repeated root x = −3; for k = −6, $(x - 3)^2 = 0$ gives x = 3. So k = 6 or k = −6.
    Excellence: reasons abstractly about repeated roots using the discriminant and verifies.

Answer key (for parents and tutors)

1. The number of texts sent by 9 students at Avondale College in one hour: 3, 5, 5, 8, 10, 12, 14, 18, 22. Find the median and the range.
   Ordered already. There are 9 values, so the median is the 5th value = 10 texts. Range = max − min = 22 − 3 = 19 texts.
   Achieved fluency: correctly locates the median and computes range.

2. Calculate the mean of these weekend earnings (in $): 40, 55, 60, 60, 85.
   Sum = 40 + 55 + 60 + 60 + 85 = 300. Mean = 300 ÷ 5 = $60.
   Achieved fluency: applies the mean formula correctly.

3. For the data 12, 15, 15, 18, 20, 21, 25, 30, find the lower quartile (Q1), upper quartile (Q3) and the interquartile range.
   8 values. Lower half: 12, 15, 15, 18 → Q1 = (15 + 15) ÷ 2 = 15. Upper half: 20, 21, 25, 30 → Q3 = (21 + 25) ÷ 2 = 23. IQR = 23 − 15 = 8.
   Achieved fluency: finds quartiles and computes IQR.

4. A dot plot of daily bus wait times (minutes) has most dots between 2 and 6, with one dot far out at 20. Describe the shape and identify any outlier.
   The distribution is right-skewed (has a tail towards the higher values). There is an outlier at 20 minutes, which is well separated from the main cluster between 2 and 6 minutes.
   Achieved fluency: describes shape and identifies an outlier.

5. A student wants to investigate reaction times at Vertex Academy. Write a suitable investigative comparison question, and state what data would need to be collected.
   Example question: 'Do students who play video games regularly tend to have faster reaction times than students who don't, at Vertex Academy?' Data needed: for a sample of students, record each person's reaction time (in milliseconds, measured with a reaction test) and whether they play video games regularly (yes/no). The reaction time is the numeric variable and the gaming group is the category used to split the two groups.
   Achieved: poses a sensible comparison question and identifies the variables/data required.

6. Given the shoe sizes of 10 netball players: 6, 7, 7, 8, 8, 8, 9, 9, 10, 11, choose a suitable display and describe the shape and centre.
   A dot plot is suitable for this single numeric variable. Median: 10 values, average of 5th and 6th = (8 + 8) ÷ 2 = 8. Mode = 8 (appears three times). The distribution is roughly symmetric, centred around size 8, ranging from 6 to 11 with no obvious outliers.
   Achieved: selects an appropriate display and describes centre and shape.

7. The times (minutes) for 12 people to complete a 3 km parkrun in New Lynn: 16, 18, 18, 19, 20, 21, 22, 23, 24, 25, 27, 30. Find the median and the IQR.
   12 values. Median = average of 6th and 7th = (21 + 22) ÷ 2 = 21.5 min. Lower half (first 6): 16, 18, 18, 19, 20, 21 → Q1 = (18 + 19) ÷ 2 = 18.5. Upper half (last 6): 22, 23, 24, 25, 27, 30 → Q3 = (24 + 25) ÷ 2 = 24.5. IQR = 24.5 − 18.5 = 6 min.
   Achieved: computes median and IQR from a straightforward data set.

8. A café records the number of coffees sold each hour: 14, 22, 18, 30, 25, 19. Calculate the mean and state whether the mean or median better represents a 'typical' hour if one hour had a huge one-off order of 90 coffees added.
   Original mean = (14 + 22 + 18 + 30 + 25 + 19) ÷ 6 = 128 ÷ 6 ≈ 21.3 coffees. If 90 is added, the data becomes seven values summing to 218, mean ≈ 31.1, pulled up by the extreme value. The median (order the seven: 14, 18, 19, 22, 25, 30, 90 → median = 22) is barely affected. So the median better represents a typical hour, because it is not distorted by the single unusually large value.
   Achieved: applies mean/median and identifies the effect of an extreme value.

9. Two Year 11 classes did the same 40-mark test. Class A: median 26, Q1 20, Q3 31. Class B: median 24, Q1 22, Q3 27. Compare the two distributions, linking centre and spread, and comment on which class did better.
   Centre: Class A's median (26) is higher than Class B's (24), so Class A students tended to score slightly higher. Spread: Class A's IQR = 31 − 20 = 11, while Class B's IQR = 27 − 22 = 5. So Class B's middle 50% of scores are much more consistent (bunched together), whereas Class A's scores are more spread out, some did very well, others

less so. Linking these: although Class A has the higher centre, the difference in medians (only 2 marks) is small compared with the spread, so I can't confidently say Class A is clearly better overall. Class A tended to score a little higher but less consistently; Class B was more reliable but slightly lower on average.

Merit: links centre and spread together to make a relational comparison.

10. A student collected commuting distances (km) from a survey posted only in the school's cycling club group chat, then concluded 'most students at our school live within 5 km of school'. Explain two problems with this conclusion and how the plan could be improved.
Problem 1, sampling bias: the sample came only from the cycling club, who are more likely to live close to school (they cycle), so it is not representative of all students. Problem 2, overgeneralising: the conclusion claims something about 'most students at our school' from a biased, self-selected sample, so the evidence doesn't support it. Improvements: use a random sample across all year levels and transport types (e.g. randomly select students from the whole school roll), and word the conclusion to match what the data actually shows, only generalising cautiously if the sampling was fair.

Merit: connects the sampling method to the validity of the conclusion.

11. Weekly pocket money ($) for a sample: 5, 10, 10, 12, 15, 15, 15, 20, 25, 60. A student says 'the average is $18.70 so a typical student gets about $19'. Using the shape of the data, evaluate this claim and suggest a better summary.
Mean = (5+10+10+12+15+15+15+20+25+60) ÷ 10 = 187 ÷ 10 = $18.70. But the data is right-skewed: nine values are $25 or under, and one large value ($60) pulls the mean up. The median = average of 5th and 6th values = (15 + 15) ÷ 2 = $15, which is below the mean. So the mean overstates the 'typical' amount. A better summary of the centre is the median ($15), because it is resistant to the high outlier and better reflects what most students actually receive.

Merit: links shape/skew of the data to the choice of an appropriate centre measure.

12. Two box plots comparing sleep hours of Year 9 and Year 13 students overlap: Year 9 median 8.5 h (IQR 7.5–9.5), Year 13 median 6.5 h (IQR 5.5–7.5). Explain, with justification, whether it is reasonable to claim that in the wider student population Year 13 students tend to sleep less than Year 9 students.
The medians differ by 2 hours (8.5 vs 6.5), which is large compared with the IQRs (both 2 hours wide). Because the shift between the two distributions is about the same size as the spread within each group, the boxes barely overlap, Year 13's upper quartile (7.5) equals Year 9's lower quartile (7.5). This clear separation means the difference is unlikely to be just sampling variation. So, provided the samples were randomly and fairly selected from each year level, it is reasonable to make a 'back to the population' claim: Year 13 students at this school do tend to sleep less than Year 9 students. The justification is that the difference in centres is large relative to the spread, giving a distinct shift rather than heavy overlap.

Excellence: justifies a population inference using the size of the shift relative to spread.

13. Prove that adding a single new value equal to the current mean of a data set leaves the mean unchanged, and explain what this shows about the mean's sensitivity.
Let the data have n values with sum S, so mean = S ÷ n. Add a new value equal to the mean, i.e. add S ÷ n. New sum = S + S/n = S(n + 1)/n. New count = n + 1. New mean = [S(n + 1)/n] ÷ (n + 1) = S/n. This equals the original mean, so the mean is unchanged. This shows the mean is a 'balance point' of the data: adding a value at the balance point doesn't shift it. It also illustrates the flip side, adding a value far from the mean shifts it noticeably, which is why the mean is sensitive to outliers while the median often is not.

Excellence: proves a general result algebraically and interprets its meaning.

14. A student surveyed 200 people at Avondale train station one Monday morning and found the median daily screen time was 1.5 hours, concluding 'New Zealanders spend about 1.5 hours per day on screens'. Critically evaluate every stage of this enquiry (problem, plan, data, analysis, conclusion) and describe a redesign that would support a valid national claim.
Problem: no clear investigative question was posed before collecting, the study seems aimed at a broad national claim, which is too ambitious for one location. Plan/Data: the sample is people at one train station, on one Monday morning, this is a convenience sample biased towards commuters and workers, and excludes non-commuters, other regions, weekends and different times, so it can't represent all New Zealanders. Analysis: only a single median is reported; there's no measure of spread or shape, and no display, so the pattern and variability are hidden. Conclusion: it generalises from a biased local sample to the whole country, which the data cannot

support. Redesign: pose a specific question (e.g. 'What is the typical daily screen time of NZ adults?'), use a large, random, nationally representative sample across regions, ages and days, collect screen time reliably (e.g. from device logs rather than recall), then analyse with a dot/box plot plus median and IQR, and only generalise to the population the sample was drawn from, acknowledging sampling variation.

Excellence: critiques the full PPDAC cycle and redesigns it to justify a valid inference.

Answer key (for parents and tutors)

1. Find 15% of $240.
   15% of $240 = 240 × 0.15 = $36.
   Achieved: applies the percentage-of-an-amount method.

2. Share $600 between two marae committees in the ratio 3 : 2.
   Total parts = 3 + 2 = 5. One part = 600 ÷ 5 = $120. First committee = 3 × 120 = $360; second = 2 × 120 = $240.
   Achieved: applies ratio sharing.

3. A car travels 180 km in 2.5 hours. Find its average speed in km/h.
   Speed = distance ÷ time = 180 ÷ 2.5 = 72 km/h.
   Achieved: calculates a rate.

4. A 2 kg bag of kūmara costs $9.60. What is the cost per kilogram?
   Cost per kg = 9.60 ÷ 2 = $4.80/kg.
   Achieved: finds a unit rate.

5. A pair of gumboots is priced at $65 before GST. GST of 15% is added at the checkout. What is the total price?
   GST = 65 × 0.15 = $9.75. Total price = 65 + 9.75 = $74.75.
   Achieved: applies percentage increase to a real context.

6. At a Pasifika festival, a food stall sells 3 items for $12. At this rate, how much would 8 items cost?
   Cost per item = 12 ÷ 3 = $4. For 8 items = 8 × 4 = $32.
   Achieved: uses a unit rate to scale a proportional problem.

7. Aroha deposits $800 in an account paying 4% simple interest per year. How much interest does she earn in 3 years?
   I = P × r × t = 800 × 0.04 × 3 = $96.
   Achieved: applies the simple interest formula.

8. A recipe for a hāngī feeds 6 people and uses 1.5 kg of pork. How much pork is needed to feed 20 people?
   Pork per person = 1.5 ÷ 6 = 0.25 kg. For 20 people = 20 × 0.25 = 5 kg.
   Achieved: uses proportional reasoning to scale a recipe.

9. A shop reduces the price of a surfboard by 20% in a sale, making the sale price $360. What was the original price, and how many dollars were saved?
   Sale price is 80% of the original, so original = 360 ÷ 0.80 = $450. Saving = 450 − 360 = $90.
   Merit: links reverse-percentage reasoning to find the original amount and the saving.

10. Two brands of coconut milk are compared. Brand A: 400 mL for $3.20. Brand B: 600 mL for $4.50. Which is better value, and by how much per litre?
    Brand A: 3.20 ÷ 0.4 = $8.00/L. Brand B: 4.50 ÷ 0.6 = $7.50/L. Brand B is better value by $8.00 − $7.50 = $0.50 per litre.
    Merit: connects unit-rate calculations to compare and justify a decision.

11. Tane is driving from Auckland to Taupō, a distance of 280 km. His car uses fuel at 7.5 L per 100 km, and petrol costs $2.80 per litre. Calculate the total fuel cost for the trip.
    Fuel used = 280 ÷ 100 × 7.5 = 2.8 × 7.5 = 21 L. Cost = 21 × 2.80 = $58.80.
    Merit: works through a multi-step problem linking rate, quantity and cost.

12. A tourist changes $500 NZD to Fijian dollars at a rate of 1 NZD = 1.35 FJD, paying a $10 NZD flat fee first. In Fiji she spends 600 FJD, then changes the rest back at 1 NZD = 1.40 FJD. How many NZD does she end up with? Explain why she loses money overall even though nothing was 'spent' on the conversions themselves.
    After the fee: 500 − 10 = $490 NZD converted. FJD received = 490 × 1.35 = 661.50 FJD. After spending 600 FJD, she has 61.50 FJD left. Converting back: 61.50 ÷ 1.40 = $43.93 NZD (to the nearest cent). She started with $500 and now holds $43.93 plus the value of 600 FJD spent. The 600 FJD spent was worth 600 ÷ 1.35 = $444.44 at the buy rate but only 600 ÷ 1.40 = $428.57 at the sell rate. The loss comes from the 'spread', the bank buys FJD from her at a worse rate (1.40) than it sold them (1.35), plus the $10 fee, so value is lost on the exchange itself, not on

purchases.

Excellence: solves an unfamiliar multi-stage problem and justifies why the exchange spread causes a loss.

13. Two towns run water tanks. Town A's tank fills at a steady 15 L per minute; Town B's fills at 12 L per minute but already holds 900 L when both start. After how many minutes do the tanks hold the same volume, and prove your answer using an equation.
Let t = minutes. Volume A = 15t. Volume B = 900 + 12t. Set equal: 15t = 900 + 12t → 3t = 900 → t = 300 minutes. Check: A = 15 × 300 = 4500 L; B = 900 + 12 × 300 = 900 + 3600 = 4500 L. ✓ The tanks hold the same volume (4500 L) after 300 minutes.
Excellence: forms and solves an equation from a rate context and verifies the generalisation.

14. A koha (donation) fund grows by the same percentage each year. After 2 years $2000 has grown to $2205. Find the annual percentage growth rate, and use it to predict the value after 3 years, explaining any assumption.
Let the multiplier be m. Then $2000 \times m^2 = 2205$, so $m^2 = 2205 \div 2000 = 1.1025$, $m = \sqrt{1.1025} = 1.05$. The rate is 5% per year. After 3 years: $2000 \times 1.05^3 = 2000 \times 1.157625 = \$2315.25$. Assumption: the growth rate stays constant at 5% each year (compounding), which may not hold if interest rates change.
Excellence: reverses a compound-growth relationship, generalises to a further year, and states an assumption.

Answer key (for parents and tutors)

1. A water tank fills at a steady rate. After 3 minutes it holds 240 litres. What is the filling rate in litres per minute?
   Rate = 240 ÷ 3 = 80 litres per minute.
   Achieved: calculates a rate from given values.

2. A jacket priced at $80 is discounted by 25%. What is the sale price?
   Discount = 25% of $80 = 0.25 × 80 = $20. Sale price = 80 − 20 = $60.
   Achieved: applies a percentage to a context.

3. A phone plan costs $15 per month plus $0.10 per text. Write a formula for the total cost C (in dollars) for t texts in a month.
   C = 15 + 0.10t.
   Achieved: expresses a linear relationship as a formula.

4. A line graph of a car's distance shows it travelled 150 km in 2 hours at a steady speed. What was its speed?
   Speed = distance ÷ time = 150 ÷ 2 = 75 km/h.
   Achieved: reads a graph and computes a rate.

5. Aroha earns $22.50 per hour at a café in Avondale. One week she works 18 hours. Using the model Pay = rate × hours, calculate her pay for the week.
   Pay = 22.50 × 18 = $405.00.
   Achieved: selects and applies a multiplication model.

6. A table shows daily rainfall in Auckland: Mon 4 mm, Tue 0 mm, Wed 12 mm, Thu 8 mm, Fri 6 mm. What was the total rainfall, and the mean daily rainfall over the five days?
   Total = 4 + 0 + 12 + 8 + 6 = 30 mm. Mean = 30 ÷ 5 = 6 mm per day.
   Achieved: reads a table and calculates total and mean.

7. A recipe for 4 people needs 600 g of flour. Kiri is cooking for 10 people. How much flour does she need?
   Flour per person = 600 ÷ 4 = 150 g. For 10 people = 150 × 10 = 1500 g (1.5 kg).
   Achieved: applies proportional reasoning to a context.

8. A courier charges a $6 base fee plus $2.50 per kilometre. Use C = 6 + 2.50d to find the cost of a 14 km delivery.
   C = 6 + 2.50 × 14 = 6 + 35 = $41.00.
   Achieved: substitutes into a given linear model.

9. Two gyms in West Auckland offer memberships. FitZone charges $40 joining fee plus $18 per month. PowerHouse charges no joining fee but $22 per month. After how many months do the two gyms cost the same amount, and what is that amount?
   FitZone: 40 + 18m. PowerHouse: 22m. Set equal: 40 + 18m = 22m, so 40 = 4m, m = 10 months. Cost = 22 × 10 = $220 (check FitZone: 40 + 18×10 = 220). They cost the same at 10 months, $220.
   Merit: links two models and solves to compare them.

10. A shop increases a $250 bike price by 20% for the summer, then in autumn reduces the new price by 20%. A customer says 'the price is back to $250'. Show whether the customer is correct.
    After increase: 250 × 1.20 = $300. After decrease: 300 × 0.80 = $240. The final price is $240, not $250, so the customer is incorrect. A 20% decrease is taken from the larger $300, so it removes more dollars than the 20% increase added.
    Merit: connects successive percentage changes and reasons about the result.

11. A graph shows a plant's height: 5 cm at week 0, growing steadily to 25 cm at week 4. Write a formula for the height h after w weeks, then use it to predict the height at week 7. State one reason the prediction may be unreliable.
    Growth per week = (25 − 5) ÷ 4 = 20 ÷ 4 = 5 cm/week. Formula: h = 5 + 5w. At week 7: h = 5 + 5×7 = 40 cm. It may be unreliable because plants do not usually keep growing at a constant rate forever, growth often slows, so extrapolating beyond the data is risky.
    Merit: connects a graph to a formula and comments on limitations.

12. A phone company offers Plan A: $30 flat per month for unlimited data. Plan B: $10 per month plus $2 per GB. For what range of data usage is Plan A the cheaper choice? Justify your answer with a general argument.
Plan B cost = 10 + 2g where g = GB used. Plan A = 30. Plan A is cheaper when 30 < 10 + 2g, i.e. 20 < 2g, so g > 10. Therefore Plan A is cheaper for any usage above 10 GB per month; at exactly 10 GB they cost the same ($30); below 10 GB Plan B is cheaper. Since Plan A's cost never changes, the more data you use the more Plan A saves you relative to B.
Excellence: solves an inequality and generalises the decision rule.

13. A claim in a local newsletter states: 'Averaging 90 km/h over a 180 km trip means you can drive 100 km/h for the first half of the distance and 80 km/h for the second half.' Determine whether this claim is true, showing your reasoning.
Split into two 90 km halves. First half at 100 km/h takes 90 ÷ 100 = 0.9 hours. Second half at 80 km/h takes 90 ÷ 80 = 1.125 hours. Total time = 2.025 hours for 180 km. Average speed = 180 ÷ 2.025 ≈ 88.9 km/h, not 90 km/h. The claim is false because average speed is total distance ÷ total time, not the mean of the two speeds, the slower half takes longer so it 'weighs' more.
Excellence: disproves a claim with correct reasoning about weighted rates.

14. A container's rental cost is modelled by C = 50 + 8d for d days. A rival offers a weekly rate of $90 per 7-day week (part weeks rounded up to a full week). Explain, with justification, for which numbers of days each option is cheaper.
Rival cost jumps in steps: 1–7 days = $90, 8–14 days = $180, and so on (90 × number of weeks, rounding up). Model C = 50 + 8d. Compare for a first week (d = 1 to 7): C ranges from 50 + 8 = $58 up to 50 + 56 = $106. The model beats $90 when 50 + 8d < 90, i.e. 8d < 40, d < 5, so for 1–4 days the formula is cheaper, and for 5–7 days the $90 week is cheaper (equal-ish near d = 5). For the second week (8–14 days) rival = $180; model = 50 + 8d, which reaches 180 when 8d = 130, d = 16.25, so across days 8–14 the model (max $162 at d=14) stays under $180, the formula is cheaper again. In general the rival's step price is only competitive late in each week; overall the formula is better except roughly days 5–7. A renter should choose the $90 week only for a 5–7 day hire.
Excellence: compares a continuous model with a step function and justifies the optimal choice across ranges.