Vertex Academy

Year 10 Maths workbook

9 worksheets · answers at the back

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1. Rational, irrational, and scientific notation
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7. Choosing a graph, scaling it, and lines of best fit
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Year 10 · Number: Powers, Roots, Ratios & Rates

Name: _____ Date: _____

Kia ora! This sheet mixes exponent rules, roots, scientific notation, percentages, ratios and rates. Show your working and keep exact answers exact where you can, no calculator needed for most questions.

Warm-up

1. Use an exponent rule to simplify and then evaluate $2^3 \times 2^4$.

2. Simplify $\sqrt{3} + \sqrt{3}$.

3. Write 0.75 as a fraction in simplest form and as a percentage.

4. Evaluate $(-2)^3$.

Practice

5. From this list, write down which numbers are irrational: $\sqrt{16}$, $\sqrt{10}$, π, $\sqrt[3]{27}$, 0.6.

6. Evaluate 3^{-2} and 5^0.

7. Write 45 600 in scientific notation to 3 significant figures.

8. Simplify $\sqrt{2} \times \sqrt{8}$, then simplify $\sqrt{50} - \sqrt{32}$.

9. At Pak'nSave in Avondale, a block of cheese went up from $12.00 to $15.00. What is the percentage increase?

10. Aria drives from Avondale to Hamilton, a distance of 120 km, in 1½ hours. What is her average speed in km/h? If 30% of the trip is on the motorway, how many kilometres is that?

11. A jacket is on sale for $68 after a 15% discount. What was the original price?

Challenge

12. Evaluate exactly, showing order of operations: $\sqrt{(3^2 + 4^2)} + (-2)^3 \times 2$.

13. Light travels at about 3.0×10^8 metres per second. The Sun is about 1.5×10^{11} metres from Earth. How long (in seconds) does sunlight take to reach Earth? Give your answer to 2 significant figures.

Year 10 · Number: Money, Rates & Growing Your Savings

Name: _____ Date: _____

Kia ora! This sheet gives you practice swapping between currencies, working out part-costs from a price per kilogram, and watching interest build up month by month. Grab a calculator and take your time.

Warm-up

1. Convert $50 NZD into Australian dollars if 1 NZD = 0.93 AUD.

2. A bag of rice costs $8.00 per kilogram. What is the cost of 400 g?

3. Find 2.5% of $100.

4. Convert 60 euros into NZD if 1 NZD = 0.56 EUR.

Practice

5. Mere is buying online from the USA. Convert $320 NZD into US dollars if 1 NZD = 0.61 USD.

6. At the New World deli, tasty cheese is priced at $22.00 per kilogram. How much would 350 g cost?

7. Tama invests $200 at 3% interest per month. Working month by month, how much does he have after 3 months?

8. Aroha is flying to Japan for a school trip. She changes $500 NZD into Japanese yen at a rate of 1 NZD = 89 yen. How many yen does she receive?

9. Rump steak at the butcher is $18.90 per kilogram. Whetu buys a 600 g piece. What does she pay?

10. Sione puts $150 into a scheme paying 2% per month. How much will he have after 4 months? (Work month by month.)

Challenge

11. At the supermarket, a 250 g block of chocolate costs $4.50, while a 400 g block costs $6.80. Which block is better value per kilogram, and by how much per kg?

12. Liam invests $500 at 1.5% per month compound interest. How much does he have after 6 months, and how much interest has he earned? (Round to the nearest cent.)

Year 10 · Algebra: Equations, Lines & Quadratics

Name: _____ Date: _____

Kia ora! This sheet pulls together the algebra you've been learning: tidying up expressions, solving equations and inequalities, working with straight lines, and getting started on quadratics. Take your time and show every step.

Warm-up

1. Collect like terms: $5a + 3b - 2a + 7b$.

2. Expand: $4(2x - 5)$.

3. Solve: $3x + 7 = 22$.

4. Factorise using a common factor: $6y + 15$.

Practice

5. Expand and simplify: $(x + 4)(x - 3)$.

6. Factorise the quadratic: $x^2 + 7x + 12$.

7. Solve the inequality and show the solution on a number line: $-2x + 5 \leq 11$.

8. Mia hires a kayak at Lake Pupuke. It costs a $12 booking fee plus $8 per hour. If she pays $52 in total, form and solve an equation to find how many hours she hired the kayak.

9. A water tank at a farm near Kumeū starts with 20 litres and is filled at a steady 15 litres per minute. Write the equation for the volume V (litres) after t minutes in the form $V = mt + c$, then find the volume after 6 minutes.

10. Solve the quadratic equation $(x - 2)(x + 5) = 0$, and state where the related graph $y = (x - 2)(x + 5)$ crosses the x-axis.

Challenge

11. A straight line passes through the points $(1, 5)$ and $(4, 14)$. Find its gradient, then find the equation of the line in the form $y = mx + c$.

12. Factorise $x^2 + 2x - 8$ by grouping, then describe how the graph of $y = x^2 + 3$ differs from the graph of $y = x^2$.

Year 10 · Measurement: Sizing It Up: Shapes, Speed & Sig Figs

Name: _____ Date: _____

Kia ora! This sheet mixes together everything from significant figures and metric prefixes to circle areas, cylinders, Pythagoras and speed. Show your working, use one extra significant figure while you calculate, then round at the very end.

Warm-up

1. Convert 3.5 km into metres.

2. Round 47.386 to 3 significant figures.

3. Find the area of a circle with radius 5 cm, to 3 significant figures.

4. Convert 2500 mg into grams.

Practice

5. Find the surface area of a rectangular prism (a shoebox) measuring 8 cm × 5 cm × 3 cm.

6. A cylinder has radius 4 cm and height 10 cm. Find its volume to 3 significant figures.

7. Word problem: Aroha drives from Avondale to Hamilton, a distance of 90 km, in 1.5 hours. What is her average speed in km/h?

8. Word problem: A cylindrical rainwater tank has radius 0.6 m and height 1.2 m. Find its capacity in litres, to 3 significant figures. (1 m^3 = 1000 L.)

9. A right-angled triangle has legs of 6 cm and 8 cm. Use Pythagoras to find the length of the hypotenuse.

10. A window is made of a rectangle 10 cm wide and 6 cm tall, topped by a semicircle of radius 5 cm. Find the total area to 3 significant figures.

Challenge

11. A triangular prism has a right-angled triangular cross-section with legs 6 cm and 8 cm, and a length of 15 cm. Find (a) its volume, and (b) its total surface area.

12. A delivery drone flies in a straight line from point (2, 3) to point (10, 9) on a map grid where each unit represents 100 m. The trip takes 20 seconds. Find (a) the straight-line distance in metres, and (b) the drone's speed in km/h.

Year 10 · Geometry: Similar & Congruent Shapes

Name: _____ Date: _____

Hi! This sheet is all about similar and congruent shapes. You'll use matching angles and proportional sides to hunt down missing lengths and angles. Take your time and show your working.

Warm-up

1. Two shapes are congruent. What can you say about their corresponding sides and angles?

2. Two shapes are similar. What can you say about their corresponding angles?

3. A triangle is enlarged by a scale factor of 3. If one side was 4 cm, how long is the matching side on the enlarged triangle?

4. Two rectangles are similar. The small one is 2 cm by 5 cm. The large one has a shortest side of 6 cm. What is the scale factor?

Practice

5. Triangle ABC is similar to triangle DEF. AB = 6 cm corresponds to DE = 9 cm. If BC = 8 cm, find the length of EF.

6. In two similar triangles, one has angles 40° and 75°. What are all three angles in the other triangle?

7. A photo 10 cm wide and 15 cm tall is enlarged so its width becomes 25 cm. If it stays similar (same shape), how tall will the enlarged photo be?

8. Aroha stands next to a flagpole at Avondale College. She is 1.6 m tall and casts a shadow 2 m long. At the same time the flagpole casts a shadow 10 m long. Using similar triangles, how tall is the flagpole?

9. Two right-angled triangles are similar. The smaller has legs 3 cm and 4 cm with hypotenuse 5 cm. The larger triangle's hypotenuse is 15 cm. Find its two legs.

10. A model of the Sky Tower is built to a scale of 1 : 400. The real Sky Tower is 328 m tall. How tall is the model, in metres and then in centimetres?

Challenge

11. In the diagram, a small triangle sits inside a larger triangle sharing the same top vertex, with the bases parallel. The small triangle has height 4 cm and base 6 cm. The large triangle has total height 10 cm (measured from the same

vertex). The two triangles are similar. Find the base of the large triangle.

12. Two similar rectangles have a length scale factor of 4. The smaller rectangle has an area of 12 cm². (a) What is the area scale factor? (b) What is the area of the larger rectangle?

Year 10 · Statistics: Sampling Smarter, Inferring Better

Name: _____ Date: _____

Hi! This sheet helps you think about how we collect good data through sampling, why random samples matter, and when to use the mean or median. Take your time and show your reasoning, statisticians explain their thinking!

Warm-up

1. What is the difference between a census and a sample?

2. In simple random sampling, what is true about each item's chance of being chosen?

3. A survey measures the height (in cm) of students, a numerical variable. What is the smallest sample size recommended for a reliable inference?

4. Name one problem that biased sampling can cause.

Practice

5. Mere wants to know the favourite lunch food of students at Avondale College (about 1,900 students). Should she use a census or a sample? Explain.

6. A student surveys only her netball team about how many hours of sport students play each week, then claims her result represents the whole school. Why is this biased, and how could she fix it?

7. Explain how you could use technology (such as a random number generator) to choose a simple random sample of 30 students from a roll of 450 students.

8. For measuring students' reaction times (a numerical variable), why is a sample of 30 considered enough, but for finding what percentage of students prefer a certain phone brand (a categorical variable) about 1,000 is recommended?

9. The weekly pocket money of 9 students (in $) is: 5, 8, 10, 10, 12, 15, 20, 25, 90. Find the mean and the median. Which is the better measure of the typical amount, and why?

10. A statistical question asks: 'What is the typical time Year 10 students take to travel to Vertex Academy?' Most students take 5–20 minutes, but a few travel over an hour. Would the mean or median better answer this question? Explain.

Challenge

11. A local café owner wants to know what proportion of Avondale residents would buy a new plant-based menu. She surveys 40 customers who are already in her café on a Sunday morning. Identify TWO sources of bias, and describe a better sampling plan including a suitable sample size.

12. Two random samples of daily rainfall (in mm) are taken in Auckland. Sample A: 0, 0, 2, 3, 5, 5, 8, 12, 15 (9 days). Sample B is the same but adds one very wet day of 60 mm (10 days). Calculate the mean and median for each sample. Explain what happens to each measure and what this shows about choosing a measure of central tendency.

Year 10 · Statistics: Picturing Data

Name: _____ Date: _____

This sheet helps you choose the right graph, pick sensible scales, and read patterns from data. You'll also draw a line of best fit to make predictions, a skill you'll use lots next year!

Warm-up

1. Name the best visualisation to show how many students at Avondale College travel to school by bus, car, bike, or walking (a category variable with counts).

2. A dot plot of shoe sizes has these dots: size 7 (2 dots), size 8 (5 dots), size 9 (3 dots), size 10 (1 dot). How many people are in the data set, and what is the most common (modal) shoe size?

3. For a scatter plot investigating whether hours studied predicts a test score, which variable goes on the x-axis and which on the y-axis?

4. A data set of daily temperatures ranges from 6°C to 27°C. Suggest a sensible scale for the vertical axis of a graph.

Practice

5. A café in Avondale records the number of flat whites sold each day for a week: Mon 42, Tue 38, Wed 55, Thu 47, Fri 63, Sat 71, Sun 29. Which visualisation best shows the pattern across the week, and what does it show?

6. Twenty Year 10 students recorded their reaction times (seconds) in an experiment. Explain why a histogram, not a bar graph, is the right choice, and describe how the bars differ.

7. Here are the maths test marks (out of 20) for two classes. Class A: 12, 14, 15, 15, 16, 18, 19. Class B: 8, 11, 13, 15, 17, 18, 20. Find the median of each class and state which class performed better on average.

8. A survey asks Year 10 students whether they own a bike (Yes/No) and whether they are Junior or Senior. Results: Junior Yes 18, Junior No 12, Senior Yes 9, Senior No 21. Draw a two-way table and find the proportion of Juniors who own a bike.

9. A student measures the height (cm) of a bean plant each week: Week 0: 2, Week 1: 5, Week 2: 9, Week 3: 14, Week 4: 20. On a scatter plot she draws a straight line of best fit that passes close to (0, 2) and (4, 20). Use this line to predict the height at Week 5.

10. A shop owner has this data linking the daily high temperature (°C) to ice creams sold: (18, 22), (20, 30), (23, 41), (25, 52), (28, 66). Describe the relationship you would see on a scatter plot and explain how a line of best fit helps.

Challenge

11. A teacher records study hours (x) and test score out of 100 (y) for six students: (1, 45), (2, 52), (3, 61), (4, 68), (5, 74), (6, 82). Draw a line of best fit, estimate its equation, and use it to predict the score for a student who studies 7 hours. Comment on how reliable this prediction is.

12. An investigation collects each Year 10 student's arm span (cm) and height (cm). Explain (a) which visualisation is appropriate and why, (b) how to choose axis scales if arm spans range 140–185 cm and heights range 145–190 cm, and (c) what a line of best fit would tell you.

Year 10 · Statistics: Reading Between the Data

Name: _____ Date: _____

Hi! This sheet helps you interpret data like a real statistician, comparing groups fairly, predicting from scatter plots, and spotting graphs that try to fool you. Take your time and always explain your thinking in context.

Warm-up

1. A scatter plot at a Mission Bay dairy shows daily temperature (from 15°C to 28°C) against ice-cream sales. If you use the graph to predict sales at 22°C, is that interpolation or extrapolation?

2. Using the same graph, you predict sales at 34°C. Is that interpolation or extrapolation, and why should you be cautious?

3. A line of best fit for that dairy is: sales = 12 × temperature + 30. Predict the number of ice creams sold at 20°C.

4. In the 75%-to-50% comparison rule, which value of Group A and which value of Group B do we compare?

Practice

5. Box plots compare Year 10 student heights at School A and School B. School A: min 150, Q1 158, median 163, Q3 168, max 178 (cm). School B: min 160, Q1 167, median 172, Q3 178, max 188 (cm). Using the 75%-to-50% rule, can you claim that School B students tend to be taller back in the population?

6. A news website shows a bar chart of two companies' yearly profit: Kea Ltd $42 million and Tui Ltd $45 million. The bar for Tui looks about twice as tall as Kea's. Explain how the graph misrepresents the data and how it should be fixed.

7. A study of kauri trees relates trunk circumference to age. The line of best fit is: age = 0.8 × circumference + 5, where circumference ranges from 50 cm to 200 cm. (a) Predict the age of a tree with circumference 120 cm. (b) Would you trust a prediction for a 400 cm tree?

8. A survey of 30 students at one Avondale school found 60% preferred rugby to football. A newspaper headline says: 'New Zealand teenagers prefer rugby.' Comment on this claim, taking uncertainty into account.

9. Two Year 10 classes sit the same maths test (out of 100). Class A: Q1 55, median 62, Q3 70. Class B: Q1 50, median 65, Q3 72. Using the 75%-to-50% rule, can you claim Class B tends to score higher in the population?

10. A pictogram about recycling shows one rubbish-bin symbol = 100 kg, but the bin drawn for 2023 is made twice as wide AND twice as tall as the 2022 bin to show 200 kg. Why does this mislead readers?

Challenge

11. A scatter plot relates weekly study hours (0 to 15 hours) to test score for Year 10 students. It shows a moderate positive relationship with line of best fit: score = 4 × hours + 40. (a) Predict the score for a student who studies 8 hours. (b) A student claims: 'If I study 30 hours, this predicts a score of 160, so I'm guaranteed top marks.' Give two reasons this reasoning is flawed.

12. A researcher wants to investigate: 'Do Year 10 students at Avondale schools who play a sport tend to have faster reaction times than those who don't?' She measures reaction times (seconds) for a sample of each group. Sport group: Q1 0.24, median 0.27, Q3 0.30. Non-sport group: Q1 0.29, median 0.33, Q3 0.37. (a) Apply the 75%-to-50% rule to compare the groups (remember: smaller time = faster). (b) Explain what elements of chance mean for how confident she can be.

Year 10 · Probability: Chance in Action: Experiment vs Theory

Name: _____ Date: _____

Hi! This sheet helps you see how the probabilities we predict on paper compare to what actually happens when we run experiments. Grab a die or a coin if you can, and keep an eye on how bigger experiments give steadier results.

Warm-up

1. A fair six-sided die is rolled once. What is the theoretical probability of rolling a number greater than 4? Give your answer as a fraction in simplest form.

2. A spinner was spun 50 times and landed on red 20 times. What is the experimental probability (relative frequency) of landing on red? Give a fraction, a decimal and a percentage.

3. A bag holds 3 red, 5 blue and 2 green marbles. One marble is drawn at random. What is P(blue)?

4. Using the same bag as the question above, what is P(not blue)?

Practice

5. Two fair coins are tossed together. (a) List all the outcomes in the sample space. (b) Find P(two heads). (c) Find P(at least one head).

6. Aria finds an old drawing pin and wonders about the chance it lands point-up. She tosses it 40 times and it lands point-up 24 times. (a) Why can Aria only find this probability by experiment, not by calculation? (b) What is her experimental probability of the pin landing point-up?

7. Tane rolled a fair die 120 times and recorded: 1 → 18, 2 → 25, 3 → 19, 4 → 22, 5 → 16, 6 → 20. (a) What is the theoretical probability of each number, and roughly how many of each would you expect in 120 rolls? (b) Compare the experimental probability of rolling a 2 with its theoretical probability. (c) Explain why the results are not exactly equal to the expected values.

8. At a Vertex Academy games night, two fair dice are rolled and the two numbers are added. (a) Systematically list or tabulate the sample space of possible totals and count the outcomes. (b) Find P(total = 8). (c) Find P(total is 10 or more).

9. A four-colour spinner (red, blue, yellow, green) is spun 80 times with these results: red 26, blue 18, yellow 14, green 22. (a) Calculate the experimental probability of each colour as a decimal. (b) Describe how you would draw a data visualisation of these results and what it would show.

10. On the way to Avondale School, Mia passes a set of traffic lights that is red 40% of the time, and independently her bus is late 15% of the time. (a) Explain why these two events are independent. (b) Find the probability that the light is red AND the bus is late. (c) Explain why you cannot simply add 0.40 and 0.15 to find P(red or late).

Challenge

11. Leo uses a spreadsheet to simulate tossing a fair coin. In 30 tosses he gets 12 heads; in 100 tosses he gets 46 heads; in 1000 tosses he gets 508 heads. (a) Find the experimental probability of heads for each set of trials. (b) The theoretical probability is 0.5. Which result is closest, and why? (c) State the Law of Large Numbers and explain what it predicts if Leo ran 100 000 tosses.

12. A bag contains 4 red and 6 blue marbles. Two marbles are drawn one after the other WITHOUT replacement. (a) Draw a tree diagram (describe the branches and their probabilities). (b) Find P(both red). (c) Find P(one of each colour).

Answer key (for parents and tutors)

1. Use an exponent rule to simplify and then evaluate $2^3 \times 2^4$.
   Product-of-exponents rule: $a^m \times a^n = a^{m+n}$, so $2^3 \times 2^4 = 2^{3+4} = 2^7$. Then $2^7 = 128$.

2. Simplify $\sqrt{3} + \sqrt{3}$.
   Only like roots add. $\sqrt{3} + \sqrt{3} = 2\sqrt{3}$ (two lots of $\sqrt{3}$).

3. Write 0.75 as a fraction in simplest form and as a percentage.
   0.75 = 75/100 = 3/4 (dividing top and bottom by 25). As a percentage, 0.75 = 75%.

4. Evaluate $(-2)^3$.
   $(-2)^3 = (-2) \times (-2) \times (-2)$. First $(-2) \times (-2) = 4$, then $4 \times (-2) = -8$. So $(-2)^3 = -8$.

5. From this list, write down which numbers are irrational: $\sqrt{16}$, $\sqrt{10}$, π, $\sqrt[3]{27}$, 0.6.
   $\sqrt{16} = 4$ (rational). $\sqrt{10}$ is not a perfect square, so it is a non-repeating infinite decimal, irrational. π is irrational. $\sqrt[3]{27} = 3$ (rational). 0.6 = 3/5 (rational). Irrational numbers: $\sqrt{10}$ and π. (General rule: a root of a whole number is rational only if the number is a perfect square/cube.)

6. Evaluate 3^{-2} and 5^0.
   Negative exponent rule: $a^{-m} = 1/a^m$, so $3^{-2} = 1/3^2 = 1/9$. Zero exponent rule: $a^0 = 1$ ($a \neq 0$), so $5^0 = 1$.

7. Write 45 600 in scientific notation to 3 significant figures.
   Move the decimal point so the first factor is between 1 and 10: 4.56×10^4 (the point moved 4 places left). The value 4.56 already has 3 significant figures, so $45\,600 = 4.56 \times 10^4$.

8. Simplify $\sqrt{2} \times \sqrt{8}$, then simplify $\sqrt{50} - \sqrt{32}$.
   $\sqrt{a} \times \sqrt{b} = \sqrt{(ab)}$: $\sqrt{2} \times \sqrt{8} = \sqrt{16} = 4$. For the second, write each as a multiple of $\sqrt{2}$: $\sqrt{50} = \sqrt{(25 \times 2)} = 5\sqrt{2}$ and $\sqrt{32} = \sqrt{(16 \times 2)} = 4\sqrt{2}$. So $\sqrt{50} - \sqrt{32} = 5\sqrt{2} - 4\sqrt{2} = \sqrt{2}$.

9. At Pak'nSave in Avondale, a block of cheese went up from $12.00 to $15.00. What is the percentage increase?
   Increase = $15.00 − $12.00 = $3.00. Percentage increase = increase ÷ original × 100 = 3 ÷ 12 × 100 = 0.25 × 100 = 25%. The price rose by 25%.

10. Aria drives from Avondale to Hamilton, a distance of 120 km, in 1½ hours. What is her average speed in km/h? If 30% of the trip is on the motorway, how many kilometres is that?
    Speed = distance ÷ time = 120 ÷ 1.5 = 80 km/h. Motorway distance = 30% of 120 km = 0.30 × 120 = 36 km.

11. A jacket is on sale for $68 after a 15% discount. What was the original price?
    After a 15% decrease, $68 is 85% of the original (100% − 15% = 85%). So 0.85 × original = 68. Original = 68 ÷ 0.85 = $80.00. Check: 15% of 80 = $12, and 80 − 12 = $68. ✓

12. Evaluate exactly, showing order of operations: $\sqrt{(3^2 + 4^2)} + (-2)^3 \times 2$.
    Use GEMA. Grouped first: inside the root, $3^2 + 4^2 = 9 + 16 = 25$, so $\sqrt{25} = 5$. Exponent: $(-2)^3 = -8$. Multiplicative: $-8 \times 2 = -16$. Additive: $5 + (-16) = -11$. So the answer is −11.

13. Light travels at about 3.0×10^8 metres per second. The Sun is about 1.5×10^{11} metres from Earth. How long (in seconds) does sunlight take to reach Earth? Give your answer to 2 significant figures.
    Time = distance ÷ speed = $(1.5 \times 10^{11}) \div (3.0 \times 10^8)$. Divide the number parts: $1.5 \div 3.0 = 0.5$. Divide the powers using the quotient rule: $10^{11} \div 10^8 = 10^{11-8} = 10^3$. So time = $0.5 \times 10^3 = 500$ seconds. To 2 significant figures, that is 5.0×10^2 seconds (about 500 s, or 8 minutes 20 seconds).

Answer key (for parents and tutors)

1. Convert $50 NZD into Australian dollars if 1 NZD = 0.93 AUD.
   50 × 0.93 = $46.50 AUD.
2. A bag of rice costs $8.00 per kilogram. What is the cost of 400 g?
   400 g = 0.4 kg. 8.00 × 0.4 = $3.20.
3. Find 2.5% of $100.
   2.5% = 0.025. 100 × 0.025 = $2.50.
4. Convert 60 euros into NZD if 1 NZD = 0.56 EUR.
   To go from euros to NZD, divide: 60 ÷ 0.56 = $107.14 (to the nearest cent).
5. Mere is buying online from the USA. Convert $320 NZD into US dollars if 1 NZD = 0.61 USD.
   320 × 0.61 = $195.20 USD.
6. At the New World deli, tasty cheese is priced at $22.00 per kilogram. How much would 350 g cost?
   350 g = 0.35 kg. 22.00 × 0.35 = $7.70.
7. Tama invests $200 at 3% interest per month. Working month by month, how much does he have after 3 months?
   Month 1: 200 × 1.03 = 206.00. Month 2: 206.00 × 1.03 = 212.18. Month 3: 212.18 × 1.03 = 218.5454 ≈ $218.55.
8. Aroha is flying to Japan for a school trip. She changes $500 NZD into Japanese yen at a rate of 1 NZD = 89 yen. How many yen does she receive?
   500 × 89 = 44,500 yen.
9. Rump steak at the butcher is $18.90 per kilogram. Whetu buys a 600 g piece. What does she pay?
   600 g = 0.6 kg. 18.90 × 0.6 = $11.34.
10. Sione puts $150 into a scheme paying 2% per month. How much will he have after 4 months? (Work month by month.)
    Month 1: 150 × 1.02 = 153.00. Month 2: 153.00 × 1.02 = 156.06. Month 3: 156.06 × 1.02 = 159.1812. Month 4: 159.1812 × 1.02 = 162.363624 ≈ $162.36.
11. At the supermarket, a 250 g block of chocolate costs $4.50, while a 400 g block costs $6.80. Which block is better value per kilogram, and by how much per kg?
    250 g block: 4.50 ÷ 0.25 = $18.00 per kg. 400 g block: 6.80 ÷ 0.40 = $17.00 per kg. The 400 g block is better value, by $18.00 – $17.00 = $1.00 per kg.
12. Liam invests $500 at 1.5% per month compound interest. How much does he have after 6 months, and how much interest has he earned? (Round to the nearest cent.)
    Month 1: 500 × 1.015 = 507.50. Month 2: 507.50 × 1.015 = 515.1125. Month 3: 515.1125 × 1.015 = 522.8391875. Month 4: 522.8392 × 1.015 = 530.681775. Month 5: 530.6818 × 1.015 = 538.642002. Month 6: 538.6420 × 1.015 = 546.721632 ≈ $546.72. Interest earned = 546.72 – 500 = $46.72.

Answer key (for parents and tutors)

1. Collect like terms: $5a + 3b - 2a + 7b$.
   Group the a-terms and the b-terms: $(5a - 2a) + (3b + 7b) = 3a + 10b$.

2. Expand: $4(2x - 5)$.
   Multiply each term inside the bracket by 4: $4 \times 2x = 8x$ and $4 \times (-5) = -20$. Answer: $8x - 20$.

3. Solve: $3x + 7 = 22$.
   Subtract 7 from both sides: $3x = 15$. Divide both sides by 3: $x = 5$.

4. Factorise using a common factor: $6y + 15$.
   The highest common factor of 6 and 15 is 3. $6y + 15 = 3(2y + 5)$.

5. Expand and simplify: $(x + 4)(x - 3)$.
   Use each term of the first bracket times each term of the second: $x \times x = x^2$, $x \times (-3) = -3x$, $4 \times x = 4x$, $4 \times (-3) = -12$. Combine: $x^2 - 3x + 4x - 12 = x^2 + x - 12$.

6. Factorise the quadratic: $x^2 + 7x + 12$.
   Find two numbers that multiply to 12 and add to 7. These are 3 and 4 ($3 \times 4 = 12$, $3 + 4 = 7$). So $x^2 + 7x + 12 = (x + 3)(x + 4)$.

7. Solve the inequality and show the solution on a number line: $-2x + 5 \leq 11$.
   Subtract 5 from both sides: $-2x \leq 6$. Divide both sides by -2 and reverse the inequality (dividing by a negative): $x \geq -3$. On a number line, draw a filled (closed) circle at -3 and shade the arrow to the right.

8. Mia hires a kayak at Lake Pupuke. It costs a $12 booking fee plus $8 per hour. If she pays $52 in total, form and solve an equation to find how many hours she hired the kayak.
   Let h = number of hours. The cost equation is $8h + 12 = 52$. Subtract 12: $8h = 40$. Divide by 8: $h = 5$. Mia hired the kayak for 5 hours.

9. A water tank at a farm near Kumeū starts with 20 litres and is filled at a steady 15 litres per minute. Write the equation for the volume V (litres) after t minutes in the form $V = mt + c$, then find the volume after 6 minutes.
   The starting amount is the y-intercept, $c = 20$. The rate of change is the gradient, $m = 15$. So $V = 15t + 20$. After 6 minutes: $V = 15 \times 6 + 20 = 90 + 20 = 110$ litres.

10. Solve the quadratic equation $(x - 2)(x + 5) = 0$, and state where the related graph $y = (x - 2)(x + 5)$ crosses the x-axis.
    By the zero product property, at least one bracket must equal 0. Either $x - 2 = 0$, giving $x = 2$, or $x + 5 = 0$, giving $x = -5$. The graph crosses the x-axis (the x-intercepts) at $(2, 0)$ and $(-5, 0)$.

11. A straight line passes through the points $(1, 5)$ and $(4, 14)$. Find its gradient, then find the equation of the line in the form $y = mx + c$.
    Gradient $m = \Delta y \div \Delta x = (14 - 5) \div (4 - 1) = 9 \div 3 = 3$. Substitute $m = 3$ and the point $(1, 5)$ into $y = mx + c$: $5 = 3 \times 1 + c$, so $5 = 3 + c$, giving $c = 2$. The equation is $y = 3x + 2$. (Check with $(4, 14)$: $3 \times 4 + 2 = 14$ ✓.)

12. Factorise $x^2 + 2x - 8$ by grouping, then describe how the graph of $y = x^2 + 3$ differs from the graph of $y = x^2$.
    Factorising by grouping: split the middle term using two numbers that multiply to -8 and add to 2, which are 4 and -2. $x^2 + 2x - 8 = x^2 + 4x - 2x - 8 = x(x + 4) - 2(x + 4) = (x - 2)(x + 4)$. For the graphs: $y = x^2$ has its lowest point (vertex) at $(0, 0)$. Adding 3 to give $y = x^2 + 3$ shifts the whole parabola up by 3 units, so its vertex is at $(0, 3)$. The shape and width stay exactly the same; only the vertical position changes.

Answer key (for parents and tutors)

1. Convert 3.5 km into metres.
   1 km = 1000 m, so 3.5 km = 3.5 × 1000 = 3500 m.

2. Round 47.386 to 3 significant figures.
   The first three significant figures are 4, 7 and 3. The next digit is 8, so round up: 47.386 → 47.4.

3. Find the area of a circle with radius 5 cm, to 3 significant figures.
   $A = \pi r^2 = \pi \times 5^2 = \pi \times 25 = 78.539\ldots \approx 78.5\ \text{cm}^2$.

4. Convert 2500 mg into grams.
   milli– means one thousandth, so 1 g = 1000 mg. 2500 ÷ 1000 = 2.5 g.

5. Find the surface area of a rectangular prism (a shoebox) measuring 8 cm × 5 cm × 3 cm.
   $SA = 2(lw + lh + wh) = 2(8\times5 + 8\times3 + 5\times3) = 2(40 + 24 + 15) = 2 \times 79 = 158\ \text{cm}^2$.

6. A cylinder has radius 4 cm and height 10 cm. Find its volume to 3 significant figures.
   $V = \pi r^2 h = \pi \times 4^2 \times 10 = \pi \times 16 \times 10 = \pi \times 160 = 502.65\ldots \approx 503\ \text{cm}^3$.

7. Word problem: Aroha drives from Avondale to Hamilton, a distance of 90 km, in 1.5 hours. What is her average speed in km/h?
   speed = distance ÷ time = 90 ÷ 1.5 = 60 km/h.

8. Word problem: A cylindrical rainwater tank has radius 0.6 m and height 1.2 m. Find its capacity in litres, to 3 significant figures. (1 m^3 = 1000 L.)
   $V = \pi r^2 h = \pi \times 0.6^2 \times 1.2 = \pi \times 0.36 \times 1.2 = \pi \times 0.432 = 1.3572\ldots\ m^3$. Convert: 1.3572 × 1000 = 1357.2 L ≈ 1360 L.

9. A right-angled triangle has legs of 6 cm and 8 cm. Use Pythagoras to find the length of the hypotenuse.
   $c^2 = a^2 + b^2 = 6^2 + 8^2 = 36 + 64 = 100$, so $c = \sqrt{100} = 10$ cm. (This is the 3–4–5 triple scaled by 2.)

10. A window is made of a rectangle 10 cm wide and 6 cm tall, topped by a semicircle of radius 5 cm. Find the total area to 3 significant figures.
    Rectangle area = $10 \times 6 = 60\ \text{cm}^2$. Semicircle area = $\frac{1}{2}\pi r^2 = \frac{1}{2} \times \pi \times 5^2 = \frac{1}{2} \times \pi \times 25 = 39.269\ldots\ \text{cm}^2$. Total = 60 + 39.269 = 99.27 ≈ 99.3 cm^2.

11. A triangular prism has a right-angled triangular cross-section with legs 6 cm and 8 cm, and a length of 15 cm. Find (a) its volume, and (b) its total surface area.
    (a) Cross-sectional area $A = \frac{1}{2} \times 6 \times 8 = 24\ \text{cm}^2$. Volume $V = Al = 24 \times 15 = 360\ \text{cm}^3$. (b) The hypotenuse = $\sqrt{(6^2 + 8^2)} = \sqrt{100} = 10$ cm. The two triangular ends: $2 \times 24 = 48\ \text{cm}^2$. The three rectangular faces have total area = perimeter × length = $(6 + 8 + 10) \times 15 = 24 \times 15 = 360\ \text{cm}^2$. Total surface area = $48 + 360 = 408\ \text{cm}^2$.

12. A delivery drone flies in a straight line from point (2, 3) to point (10, 9) on a map grid where each unit represents 100 m. The trip takes 20 seconds. Find (a) the straight-line distance in metres, and (b) the drone's speed in km/h.
    (a) Using the distance formula: $d = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]} = \sqrt{[(10 - 2)^2 + (9 - 3)^2]} = \sqrt{[8^2 + 6^2]} = \sqrt{(64 + 36)} = \sqrt{100} =$ 10 grid units. Each unit is 100 m, so distance = 10 × 100 = 1000 m. (b) speed = distance ÷ time = 1000 ÷ 20 = 50 m/s. Convert: 50 m/s × 3600 s = 180 000 m/h = 180 km/h.

Answer key (for parents and tutors)

1. Two shapes are congruent. What can you say about their corresponding sides and angles?
   Congruent shapes are identical in shape and size, so all corresponding sides are equal in length AND all corresponding angles are equal.

2. Two shapes are similar. What can you say about their corresponding angles?
   In similar shapes, all corresponding angles are equal. (The corresponding sides are proportional, the same scale factor, but not necessarily equal.)

3. A triangle is enlarged by a scale factor of 3. If one side was 4 cm, how long is the matching side on the enlarged triangle?
   Multiply by the scale factor: 4 cm × 3 = 12 cm.

4. Two rectangles are similar. The small one is 2 cm by 5 cm. The large one has a shortest side of 6 cm. What is the scale factor?
   Scale factor = new length ÷ old length = 6 ÷ 2 = 3. So the scale factor is 3.

5. Triangle ABC is similar to triangle DEF. AB = 6 cm corresponds to DE = 9 cm. If BC = 8 cm, find the length of EF.
   Scale factor = DE ÷ AB = 9 ÷ 6 = 1.5. So EF = BC × 1.5 = 8 × 1.5 = 12 cm.

6. In two similar triangles, one has angles 40° and 75°. What are all three angles in the other triangle?
   The third angle in the first triangle is 180° − 40° − 75° = 65°. Since the triangles are similar, corresponding angles are equal, so the other triangle also has angles of 40°, 75° and 65°.

7. A photo 10 cm wide and 15 cm tall is enlarged so its width becomes 25 cm. If it stays similar (same shape), how tall will the enlarged photo be?
   Scale factor = 25 ÷ 10 = 2.5. New height = 15 × 2.5 = 37.5 cm.

8. Aroha stands next to a flagpole at Avondale College. She is 1.6 m tall and casts a shadow 2 m long. At the same time the flagpole casts a shadow 10 m long. Using similar triangles, how tall is the flagpole?
   The triangles (person + shadow, and pole + shadow) are similar because the sun's rays make equal angles. Scale factor = 10 ÷ 2 = 5. Flagpole height = 1.6 × 5 = 8 m.

9. Two right-angled triangles are similar. The smaller has legs 3 cm and 4 cm with hypotenuse 5 cm. The larger triangle's hypotenuse is 15 cm. Find its two legs.
   Scale factor = 15 ÷ 5 = 3. The legs are 3 × 3 = 9 cm and 4 × 3 = 12 cm.

10. A model of the Sky Tower is built to a scale of 1 : 400. The real Sky Tower is 328 m tall. How tall is the model, in metres and then in centimetres?
    Model height = 328 ÷ 400 = 0.82 m. In centimetres: 0.82 × 100 = 82 cm.

11. In the diagram, a small triangle sits inside a larger triangle sharing the same top vertex, with the bases parallel. The small triangle has height 4 cm and base 6 cm. The large triangle has total height 10 cm (measured from the same vertex). The two triangles are similar. Find the base of the large triangle.
    Because the bases are parallel, the two triangles are similar (equal corresponding angles). Scale factor = large height ÷ small height = 10 ÷ 4 = 2.5. Large base = 6 × 2.5 = 15 cm.

12. Two similar rectangles have a length scale factor of 4. The smaller rectangle has an area of 12 cm². (a) What is the area scale factor? (b) What is the area of the larger rectangle?
    (a) Area scale factor = (length scale factor)² = 4² = 16. (b) Larger area = 12 × 16 = 192 cm². Key idea: when lengths scale by k, areas scale by k², a stepping stone into Year 11 work on area and volume of similar shapes.

Answer key (for parents and tutors)

1. What is the difference between a census and a sample?
   A census collects data from every member of the population, while a sample collects data from only a selected part of the population. A sample is used to make inferences about the whole population when a census is impractical.
2. In simple random sampling, what is true about each item's chance of being chosen?
   Every item in the population has an equal probability (an equal chance) of being selected. This is what makes the sample fair and free from bias.
3. A survey measures the height (in cm) of students, a numerical variable. What is the smallest sample size recommended for a reliable inference?
   At least 30. For numerical variables, a sample of at least 30 is recommended to make a reliable inference about the population.
4. Name one problem that biased sampling can cause.
   Biased sampling leads to data that does not fairly represent the population, producing inaccurate and misleading statistics (for example, over-representing one group and drawing a wrong conclusion).
5. Mere wants to know the favourite lunch food of students at Avondale College (about 1,900 students). Should she use a census or a sample? Explain.
   A sample. Surveying all 1,900 students would take too much time and effort, so a well-chosen sample can give a reliable estimate. Favourite lunch food is a categorical variable, so she should aim for a large sample (about 1,000) collected using technology, or at least the largest practical random sample.
6. A student surveys only her netball team about how many hours of sport students play each week, then claims her result represents the whole school. Why is this biased, and how could she fix it?
   It is biased because netball players are more likely to play a lot of sport than the average student, so the team is not representative of all students. To fix it, she should take a simple random sample from the entire school roll, giving every student an equal chance of selection.
7. Explain how you could use technology (such as a random number generator) to choose a simple random sample of 30 students from a roll of 450 students.
   Number every student on the roll from 1 to 450. Use a random number generator to produce 30 different whole numbers between 1 and 450 (ignoring any repeats). The students matching those numbers form the simple random sample. Because each number is equally likely, each student has an equal chance of being chosen.
8. For measuring students' reaction times (a numerical variable), why is a sample of 30 considered enough, but for finding what percentage of students prefer a certain phone brand (a categorical variable) about 1,000 is recommended?
   Numerical variables give a summary such as a mean or median, and a sample of at least 30 is usually enough to estimate these reliably. Categorical variables report proportions or percentages, which vary more from sample to sample, so a much larger sample (about 1,000, obtained using technology) is needed to estimate a percentage reliably.
9. The weekly pocket money of 9 students (in $) is: 5, 8, 10, 10, 12, 15, 20, 25, 90. Find the mean and the median. Which is the better measure of the typical amount, and why?
   Mean = (5+8+10+10+12+15+20+25+90) ÷ 9 = 195 ÷ 9 ≈ $21.67. Median = the 5th value (middle of 9) = $12. The median ($12) is the better measure because the $90 value is an outlier that pulls the mean upward, making $21.67 higher than what most students actually receive. The median is not affected by the outlier, so it better represents the typical amount.
10. A statistical question asks: 'What is the typical time Year 10 students take to travel to Vertex Academy?' Most students take 5–20 minutes, but a few travel over an hour. Would the mean or median better answer this question? Explain.
    The median would be better. The few very long travel times are outliers that would inflate the mean, making the 'typical' time seem longer than it is for most students. The median is resistant to these extreme values and gives a fairer picture of the typical travel time.

11. A local café owner wants to know what proportion of Avondale residents would buy a new plant-based menu. She surveys 40 customers who are already in her café on a Sunday morning. Identify TWO sources of bias, and describe a better sampling plan including a suitable sample size.
Source 1: The sample only includes people already in her café, who are more likely to enjoy her food and may not represent all Avondale residents. Source 2: Sunday-morning customers may differ from the wider population (for example, in age, diet, or income), so the timing adds further bias. Better plan: The variable (would buy / would not buy) is categorical, so she should aim for about 1,000 responses obtained using technology. She could obtain a list or numbering of Avondale residents and use a random number generator to select participants (simple random sampling), so every resident has an equal chance of being chosen. This removes the café-only and timing bias and gives a large enough sample for a reliable percentage estimate.

12. Two random samples of daily rainfall (in mm) are taken in Auckland. Sample A: 0, 0, 2, 3, 5, 5, 8, 12, 15 (9 days). Sample B is the same but adds one very wet day of 60 mm (10 days). Calculate the mean and median for each sample. Explain what happens to each measure and what this shows about choosing a measure of central tendency.
Sample A: Mean = (0+0+2+3+5+5+8+12+15) ÷ 9 = 50 ÷ 9 ≈ 5.6 mm. Median = 5th value = 5 mm. Sample B: Sum = 50 + 60 = 110, Mean = 110 ÷ 10 = 11 mm. Ordered data: 0,0,2,3,5,5,8,12,15,60; median = average of 5th and 6th values = (5+5) ÷ 2 = 5 mm. Result: Adding the 60 mm day nearly doubled the mean (from 5.6 to 11 mm) but left the median unchanged at 5 mm. This shows the mean is strongly affected by outliers while the median is resistant. For rainfall data with occasional extreme wet days, the median often better represents the typical daily rainfall, though the mean is useful if you need a total (for example, total rainfall = mean × number of days).

Answer key (for parents and tutors)

1. Name the best visualisation to show how many students at Avondale College travel to school by bus, car, bike, or walking (a category variable with counts).
   A bar graph. The four travel methods are categories, and we want to compare their frequencies. Bars (with gaps between them) let you compare category counts at a glance. A frequency table would also record the counts, but a bar graph best SHOWS the comparison.

2. A dot plot of shoe sizes has these dots: size 7 (2 dots), size 8 (5 dots), size 9 (3 dots), size 10 (1 dot). How many people are in the data set, and what is the most common (modal) shoe size?
   Total people = 2 + 5 + 3 + 1 = 11. The modal shoe size is 8, because it has the most dots (5), which is the highest frequency.

3. For a scatter plot investigating whether hours studied predicts a test score, which variable goes on the x-axis and which on the y-axis?
   Hours studied is the explanatory (predictive/independent) variable, so it goes on the x-axis. Test score is the response (dependent) variable, so it goes on the y-axis. The predictor always goes on x.

4. A data set of daily temperatures ranges from 6°C to 27°C. Suggest a sensible scale for the vertical axis of a graph.
   Start the axis at 0°C and go up to 30°C, marking every 5°C (0, 5, 10, 15, 20, 25, 30). This covers the full range (6°C to 27°C) with even, easy-to-read intervals and clear units (°C).

5. A café in Avondale records the number of flat whites sold each day for a week: Mon 42, Tue 38, Wed 55, Thu 47, Fri 63, Sat 71, Sun 29. Which visualisation best shows the pattern across the week, and what does it show?
   A time-series graph (points joined in day order) is best because the data is measured over time. Plot day on the x-axis and flat whites sold on the y-axis (scale 0 to 80, marking every 10). The pattern shows sales generally rising through the week, peaking on Saturday (71), then dropping sharply on Sunday (29).

6. Twenty Year 10 students recorded their reaction times (seconds) in an experiment. Explain why a histogram, not a bar graph, is the right choice, and describe how the bars differ.
   Reaction time is a numerical (continuous) variable, so the values are grouped into intervals (e.g. 0.20–0.29 s, 0.30–0.39 s, …). A histogram is used for grouped numerical data and its bars TOUCH (no gaps), showing a continuous number line. A bar graph is only for categories and has gaps between bars.

7. Here are the maths test marks (out of 20) for two classes. Class A: 12, 14, 15, 15, 16, 18, 19. Class B: 8, 11, 13, 15, 17, 18, 20. Find the median of each class and state which class performed better on average.
   Each class has 7 values, so the median is the 4th value when ordered. Class A ordered: 12, 14, 15, 15, 16, 18, 19 → median = 15. Class B ordered: 8, 11, 13, 15, 17, 18, 20 → median = 15. Both medians are 15, so the classes performed the same 'on average' by the median. (Class B is more spread out: range 20−8 = 12 vs Class A's 19−12 = 7.)

8. A survey asks Year 10 students whether they own a bike (Yes/No) and whether they are Junior or Senior. Results: Junior Yes 18, Junior No 12, Senior Yes 9, Senior No 21. Draw a two-way table and find the proportion of Juniors who own a bike.
   Two-way table: Junior: Yes 18, No 12, Total 30 Senior: Yes 9, No 21, Total 30 Totals: Yes 27, No 33, Grand total 60. Proportion of Juniors who own a bike = 18 ÷ 30 = 0.6, which is 60%.

9. A student measures the height (cm) of a bean plant each week: Week 0: 2, Week 1: 5, Week 2: 9, Week 3: 14, Week 4: 20. On a scatter plot she draws a straight line of best fit that passes close to (0, 2) and (4, 20). Use this line to predict the height at Week 5.
   Find the slope of the line through (0, 2) and (4, 20): slope = (20 − 2) ÷ (4 − 0) = 18 ÷ 4 = 4.5 cm per week. The line's equation is height = 2 + 4.5 × week. At Week 5: height = 2 + 4.5 × 5 = 2 + 22.5 = 24.5 cm. So predict about 24–25 cm. (Note: real growth may curve, so this is an estimate.)

10. A shop owner has this data linking the daily high temperature (°C) to ice creams sold: (18, 22), (20, 30), (23, 41), (25, 52), (28, 66). Describe the relationship you would see on a scatter plot and explain how a line of best fit helps.
    Plot temperature on the x-axis (explanatory) and ice creams sold on the y-axis (response). The points rise from lower-left to upper-right, showing a strong positive relationship: as temperature increases, ice cream sales

increase. An eyeballed line of best fit drawn through the middle of the points lets the owner predict sales for temperatures not measured, for example, reading up from 22°C gives a prediction of roughly 36–38 ice creams.

11. A teacher records study hours (x) and test score out of 100 (y) for six students: (1, 45), (2, 52), (3, 61), (4, 68), (5, 74), (6, 82). Draw a line of best fit, estimate its equation, and use it to predict the score for a student who studies 7 hours. Comment on how reliable this prediction is.
Choose two points the line passes near, e.g. (1, 45) and (6, 82). Slope = (82 − 45) ÷ (6 − 1) = 37 ÷ 5 = 7.4 marks per hour. Using point (1, 45): y = 45 + 7.4(x − 1) = 7.4× + 37.6. Check at x = 6: 7.4×6 + 37.6 = 44.4 + 37.6 = 82 ✓. Predict for x = 7: y = 7.4 × 7 + 37.6 = 51.8 + 37.6 = 89.4, so about 89 marks. Reliability: 7 hours is just beyond the data range (1–6 hours), so this is extrapolation and less reliable, scores also can't exceed 100, and the real relationship may flatten off.

12. An investigation collects each Year 10 student's arm span (cm) and height (cm). Explain (a) which visualisation is appropriate and why, (b) how to choose axis scales if arm spans range 140–185 cm and heights range 145–190 cm, and (c) what a line of best fit would tell you.
(a) A scatter plot is appropriate because both variables are numerical and we are investigating a RELATIONSHIP between paired data (each student gives one point). (b) Don't start at 0 (the values cluster far from zero). For the x-axis (arm span) start at 140 cm and go to 185 cm marking every 5 cm; for the y-axis (height) start at 145 cm and go to 190 cm marking every 5 cm. Even 5 cm intervals spread the points out clearly and use the graph space well. (c) A line of best fit summarises the trend, for arm span vs height we'd expect a strong positive relationship (taller people tend to have wider arm spans, roughly equal to their height), so the line would rise steadily and let us predict a likely height for a given arm span.

Answer key (for parents and tutors)

1. A scatter plot at a Mission Bay dairy shows daily temperature (from 15°C to 28°C) against ice-cream sales. If you use the graph to predict sales at 22°C, is that interpolation or extrapolation?
   Interpolation. 22°C lies inside the range of the temperature data (15°C to 28°C), so the prediction is within the data and is called an interpolation.

2. Using the same graph, you predict sales at 34°C. Is that interpolation or extrapolation, and why should you be cautious?
   Extrapolation, because 34°C is outside the data range (15°C–28°C). We have no evidence the same pattern continues past 28°C, so the prediction is less reliable.

3. A line of best fit for that dairy is: sales = 12 × temperature + 30. Predict the number of ice creams sold at 20°C.
   sales = 12 × 20 + 30 = 240 + 30 = 270 ice creams.

4. In the 75%-to-50% comparison rule, which value of Group A and which value of Group B do we compare?
   We compare the upper quartile (Q3, the 75% mark) of Group A with the median (the 50% mark) of Group B. If Group B's median is larger than Group A's upper quartile, then more than 50% of B is larger than 75% of A.

5. Box plots compare Year 10 student heights at School A and School B. School A: min 150, Q1 158, median 163, Q3 168, max 178 (cm). School B: min 160, Q1 167, median 172, Q3 178, max 188 (cm). Using the 75%-to-50% rule, can you claim that School B students tend to be taller back in the population?
   Compare School B's median (172 cm) with School A's upper quartile Q3 (168 cm). Since 172 > 168, more than 50% of B's data is larger than 75% of A's data. The rule is satisfied, so we CAN make the claim that School B students tend to be taller than School A students back in the population. (We still note there is overlap and variability, so it is a tendency, not a certainty.)

6. A news website shows a bar chart of two companies' yearly profit: Kea Ltd $42 million and Tui Ltd $45 million. The bar for Tui looks about twice as tall as Kea's. Explain how the graph misrepresents the data and how it should be fixed.
   The vertical axis does not start at 0 (it likely starts around $40m). This 'truncated axis' exaggerates the small difference: $45m is only about 7% more than $42m, not double. To represent the data fairly, the y-axis should start at $0, which would show two bars of nearly the same height.

7. A study of kauri trees relates trunk circumference to age. The line of best fit is: age = 0.8 × circumference + 5, where circumference ranges from 50 cm to 200 cm. (a) Predict the age of a tree with circumference 120 cm. (b) Would you trust a prediction for a 400 cm tree?
   (a) age = 0.8 × 120 + 5 = 96 + 5 = 101 years. This is an interpolation (120 cm is inside 50–200 cm), so it is reasonably reliable. (b) A 400 cm circumference is far outside the data range (extrapolation), so the prediction would be unreliable, the growth pattern may change for very old trees, and we have no data there.

8. A survey of 30 students at one Avondale school found 60% preferred rugby to football. A newspaper headline says: 'New Zealand teenagers prefer rugby.' Comment on this claim, taking uncertainty into account.
   The claim over-reaches. The data comes from only 30 students at ONE school, which is a small, non-random sample and may not represent all NZ teenagers. Because of sampling variability (a different sample could give a different result), we cannot confidently generalise to the whole population. A fairer statement is that in this sample, more students preferred rugby, but a larger, more representative survey would be needed to make a claim about all NZ teenagers.

9. Two Year 10 classes sit the same maths test (out of 100). Class A: Q1 55, median 62, Q3 70. Class B: Q1 50, median 65, Q3 72. Using the 75%-to-50% rule, can you claim Class B tends to score higher in the population?
   Compare Class B's median (65) with Class A's upper quartile Q3 (70). Since 65 is NOT greater than 70, less than 50% of B's data lies above 75% of A's data. The rule is not satisfied, so we cannot claim that Class B tends to score higher back in the population. The two groups' scores are too similar to call.

10. A pictogram about recycling shows one rubbish-bin symbol = 100 kg, but the bin drawn for 2023 is made twice as wide AND twice as tall as the 2022 bin to show 200 kg. Why does this mislead readers?
    Doubling the amount should double the area, but making the picture twice as wide and twice as tall makes the 2023 bin 2 × 2 = 4 times the area of the 2022 bin. The eye compares areas, so 200 kg looks like 4 times 100 kg

instead of 2 times. The symbols should only double in one direction (or use two identical bin symbols) so the visual matches the actual doubling.

11. A scatter plot relates weekly study hours (0 to 15 hours) to test score for Year 10 students. It shows a moderate positive relationship with line of best fit: score = 4 × hours + 40. (a) Predict the score for a student who studies 8 hours. (b) A student claims: 'If I study 30 hours, this predicts a score of 160, so I'm guaranteed top marks.' Give two reasons this reasoning is flawed.
(a) score = 4 × 8 + 40 = 32 + 40 = 72. This is an interpolation (8 hours is within 0–15), so it is reasonably reliable. (b) Reason 1: 30 hours is far outside the data range (0–15), so it is an extrapolation and the linear pattern is unlikely to continue. Reason 2: the model gives 160, but scores are capped at 100, which is impossible, showing the line cannot be extended that far. Also, the relationship is only 'moderate', so there is scatter/variability and study hours do not perfectly determine the score. The prediction is not a guarantee.

12. A researcher wants to investigate: 'Do Year 10 students at Avondale schools who play a sport tend to have faster reaction times than those who don't?' She measures reaction times (seconds) for a sample of each group. Sport group: Q1 0.24, median 0.27, Q3 0.30. Non-sport group: Q1 0.29, median 0.33, Q3 0.37. (a) Apply the 75%-to-50% rule to compare the groups (remember: smaller time = faster). (b) Explain what elements of chance mean for how confident she can be.
(a) Faster means a SMALLER time, so we test whether the sport group tends to be faster (smaller). Compare the sport group's median (0.27) with the non-sport group's LOWER quartile Q1 (0.29). Since 0.27 < 0.29, more than 50% of the sport group's times are smaller (faster) than 75% of the non-sport group's times. So we CAN claim that sporty students tend to have faster reaction times back in the population. (b) Because the data comes from a sample, chance affects which students were chosen and how they performed on the day (tiredness, guessing, distraction). A different sample could give slightly different quartiles. So she should state her finding as a tendency 'back in the population', acknowledging variability, rather than as a certain fact, and note that a larger, well-chosen sample would strengthen the conclusion.

Answer key (for parents and tutors)

1. A fair six-sided die is rolled once. What is the theoretical probability of rolling a number greater than 4? Give your answer as a fraction in simplest form.
   Numbers greater than 4 are 5 and 6, so 2 favourable outcomes out of 6 possible. P(more than 4) = 2/6 = 1/3.

2. A spinner was spun 50 times and landed on red 20 times. What is the experimental probability (relative frequency) of landing on red? Give a fraction, a decimal and a percentage.
   Experimental probability = number of reds ÷ total trials = 20/50. Simplify: 20/50 = 2/5 = 0.4 = 40%.

3. A bag holds 3 red, 5 blue and 2 green marbles. One marble is drawn at random. What is P(blue)?
   Total marbles = 3 + 5 + 2 = 10. Blue marbles = 5. P(blue) = 5/10 = 1/2.

4. Using the same bag as the question above, what is P(not blue)?
   P(not blue) = 1 − P(blue) = 1 − 1/2 = 1/2. (Check: 3 red + 2 green = 5 non-blue out of 10 = 5/10 = 1/2.)

5. Two fair coins are tossed together. (a) List all the outcomes in the sample space. (b) Find P(two heads). (c) Find P(at least one head).
   (a) Sample space = {HH, HT, TH, TT}, 4 equally likely outcomes. (b) Only HH gives two heads, so P(two heads) = 1/4. (c) Outcomes with at least one head are HH, HT, TH = 3 outcomes, so P(at least one head) = 3/4.

6. Aria finds an old drawing pin and wonders about the chance it lands point-up. She tosses it 40 times and it lands point-up 24 times. (a) Why can Aria only find this probability by experiment, not by calculation? (b) What is her experimental probability of the pin landing point-up?
   (a) A drawing pin is not a regular, symmetrical shape, so the two ways it can land (point-up or point-down) are not equally likely. There is no way to work out the theoretical probability just by counting equal outcomes, so a physical experiment is the only way to explore it. (b) Experimental P(point-up) = 24/40 = 3/5 = 0.6 = 60%.

7. Tane rolled a fair die 120 times and recorded: 1 → 18, 2 → 25, 3 → 19, 4 → 22, 5 → 16, 6 → 20. (a) What is the theoretical probability of each number, and roughly how many of each would you expect in 120 rolls? (b) Compare the experimental probability of rolling a 2 with its theoretical probability. (c) Explain why the results are not exactly equal to the expected values.
   (a) Each face is equally likely, so theoretical probability = 1/6 for every number. Expected count = 120 × 1/6 = 20 of each. (b) Experimental P(2) = 25/120 ≈ 0.208, while theoretical P(2) = 1/6 ≈ 0.167. The 2 came up a bit more often than expected. (c) Because each roll is random, results from repeated trials naturally vary around the expected values, you rarely get exactly 20 of each in 120 rolls. This variation is normal chance behaviour.

8. At a Vertex Academy games night, two fair dice are rolled and the two numbers are added. (a) Systematically list or tabulate the sample space of possible totals and count the outcomes. (b) Find P(total = 8). (c) Find P(total is 10 or more).
   (a) There are 6 × 6 = 36 equally likely outcomes (first die, second die). The totals table shows: total 2 (1 way), 3 (2), 4 (3), 5 (4), 6 (5), 7 (6), 8 (5), 9 (4), 10 (3), 11 (2), 12 (1). These add to 36. (b) Total 8 happens 5 ways (2+6, 3+5, 4+4, 5+3, 6+2), so P(8) = 5/36. (c) Totals of 10, 11 or 12 have 3 + 2 + 1 = 6 ways, so P(10 or more) = 6/36 = 1/6.

9. A four-colour spinner (red, blue, yellow, green) is spun 80 times with these results: red 26, blue 18, yellow 14, green 22. (a) Calculate the experimental probability of each colour as a decimal. (b) Describe how you would draw a data visualisation of these results and what it would show.
   (a) P(red) = 26/80 = 0.325; P(blue) = 18/80 = 0.225; P(yellow) = 14/80 = 0.175; P(green) = 22/80 = 0.275. (These add to 1.0, a good check.) (b) A bar graph works well: put the four colours along the horizontal axis and either frequency (or relative frequency) up the vertical axis, drawing a bar of the matching height for each colour. The graph would show red as the tallest bar and yellow the shortest, describing the distribution of observed outcomes, the spinner appears roughly even but red came up most and yellow least in this experiment.

10. On the way to Avondale School, Mia passes a set of traffic lights that is red 40% of the time, and independently her bus is late 15% of the time. (a) Explain why these two events are independent. (b) Find the probability that the light is red AND the bus is late. (c) Explain why you cannot simply add 0.40 and 0.15 to find P(red or late).
    (a) The colour of the traffic light has no effect on whether the bus is late (they have unrelated causes), so knowing one outcome does not change the probability of the other, that is what independent means. (b) For independent

events, multiply: P(red and late) = 0.40 × 0.15 = 0.06 = 6%. (c) Adding would double-count the outcomes where the light is red AND the bus is late. The correct rule is P(red or late) = 0.40 + 0.15 − 0.06 = 0.49, subtracting the overlap so it is only counted once.

11. Leo uses a spreadsheet to simulate tossing a fair coin. In 30 tosses he gets 12 heads; in 100 tosses he gets 46 heads; in 1000 tosses he gets 508 heads. (a) Find the experimental probability of heads for each set of trials. (b) The theoretical probability is 0.5. Which result is closest, and why? (c) State the Law of Large Numbers and explain what it predicts if Leo ran 100 000 tosses.
(a) 30 tosses: 12/30 = 0.40. 100 tosses: 46/100 = 0.46. 1000 tosses: 508/1000 = 0.508. (b) The 1000-toss result (0.508) is closest to 0.5. With more trials, one-off lucky or unlucky runs have less effect on the overall relative frequency, so the estimate settles nearer the true value. (c) The Law of Large Numbers states that as the number of trials increases, the experimental probability gets closer and closer to the theoretical probability. So with 100 000 tosses Leo's proportion of heads would almost certainly be extremely close to 0.5, much closer than any of his smaller experiments.

12. A bag contains 4 red and 6 blue marbles. Two marbles are drawn one after the other WITHOUT replacement. (a) Draw a tree diagram (describe the branches and their probabilities). (b) Find P(both red). (c) Find P(one of each colour).
(a) First draw branches: red with probability 4/10 and blue with probability 6/10. If red was drawn first, 9 marbles remain (3 red, 6 blue), so second-draw branches are red 3/9 and blue 6/9. If blue was drawn first, 9 remain (4 red, 5 blue), so second-draw branches are red 4/9 and blue 5/9. The probabilities change on the second draw because the events are dependent (no replacement). (b) P(both red) = 4/10 × 3/9 = 12/90 = 2/15. (c) P(one of each) = P(red then blue) + P(blue then red) = (4/10 × 6/9) + (6/10 × 4/9) = 24/90 + 24/90 = 48/90 = 8/15.